Lambert's theorem for Kepler and Hooke on constant curvature spaces

Alain Albouy, Observatoire de Paris
E-mail address: Alain.Albouy@obspm.fr.

We will present a few results, obtained in collaboration with Zhao Lei.

1. Lambert's theorem remains true without any change in statement if we "remove Euclid's fifth postulate", i.e., if we consider the Kepler problem on a constant curvature space.

2. There is a definition of "a Lambert's theorem" such that: If a natural system has a Lambert's theorem and if its "transformation" by Appell's projection is a natural system, then the transformed system has a Lambert's theorem.

3. The harmonic oscillator in the plane (the Hooke problem) has a Lambert's theorem.

4. The second type of "transformation of the equations of dynamics" in Painlevé's theory also preserves Lambert's theorems.

(conformal transformations due to Goursat and Darboux in 1889, generalizing the $z \mapsto z^2$ map which sends Hooke on Kepler)

5. There is no Goursat-Darboux (conformal) transformation from the Kepler problem on the sphere to a natural problem on a constant curvature space.

6. The Hooke problem on the sphere, the Kepler and Hooke problems on the pseudo-sphere correspond one to each other by a Goursat-Darboux transformation (essentially made of $z \mapsto z^2$ and of stereographic projections).

This research began with the discovery of new proofs of the classical Lambert theorem [1].

References

[1] A. Albouy, *Lambert's theorem through an affine lens*, arxiv.org/abs/1711.03049.

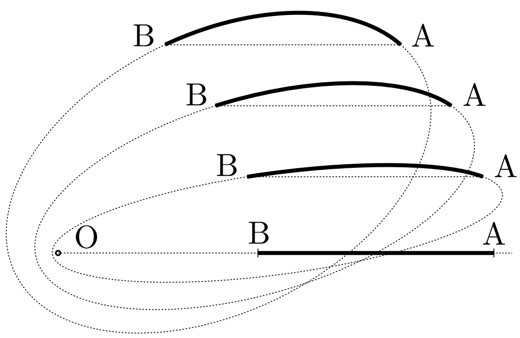

Figure 1. Keplerian arcs satisfying Lambert's conditions