Workshop on Geometry: Multiple Perspectives on Geometric Inequalities

Centre de Recerca Matemàtica

June 3rd – 7th, 2019

Organizing Committee

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1. Schedule

Monday, 3rd of June 2019	
08:30–09:00	Registration – Welcome
09:00–09:50	Alberto Abbondandolo, Ruhr-Universität *A symplectic approach to systolic inequalities: spheres of revolution*
10:00–10:50	Alexander Nabutovsky, University of Toronto *Filling metric spaces*
11:00–11:25	Coffee Break
11:25–12:15	Bram Petri, Universität Bonn *Universal properties of random surfaces*
12:20–13:50	Lunch
14:00–14:50	Alina Stancu, Concordia University *Some inequalities obtained via a curvature flow*
15:00–15:20	Coffee Break
15:30–16:20	Roman Karasev, Moscow Institute of Physics and Technology *Mahler's conjecture for some hyperplane sections*
Tuesday, 4th of June 2019	
09:00–09:50	Gabriele Benedetti, Heidelberg University *A magnetic systolic inequality on surfaces*
10:00–10:50	Panagiotis Papazoglou, University of Oxford *Isoperimetric Profiles of Surfaces and some related ques-tions in systolic and coarse geometry*
11:00–11:25	Coffee Break
11:25–12:15	Federica Fanoni, IRMA *Big mapping class groups acting on homology*
12:20–13:50	Lunch
14:00–14:50	Joseph Fu, University of Georgia *Valuations in Riemannian geometry*
15:00–15:20	Coffee Break
15:30–16:20	Poster Session

Wednesday, 5th of June 2019	
09:00–09:50	Umberto Hryniewicz, RWTH Aachen *Symplectic methods for sharp systolic inequalities*
10:00–10:50	Franz Schuster, Technische Universität Wien *Affine quermassintegrals and Minkowski valuations*
11:00–11:25	Coffee Break
11:25–12:15	Jeff Brock, Brown University
12:20–13:50	Lunch
Thursday, 6th of June 2019	
09:00–09:50	Jean Gutt, Universität zu Köln *Two periodic Reeb orbits*
10:00–10:50	Anna Siffert, Max Planck Institute for Mathematics *Large genus minimal surfaces in positive Ricci curvature*
11:00–11:25	Coffee Break
11:25–12:15	Juan Souto, Université de Rennes *Counting curves of a given type, revisited*
12:20–13:50	Lunch
14:00–14:50	Dmitry Faifman, Université de Montréal *Intrinsic volumes in contact and pseudo-Riemannian geometries*
15:00–15:20	Coffee Break
15:30–16:20	Maxime Fortier Bourque, University of Glasgow *Sublinearly many systoles that fill*

Friday, 7th of June 2019	
10:00–10:50	Marco Mazzucchelli, École Normale Supérieure de Lyon MIN-MAX *characterizations of besse and zoll reeb flows*
11:00–11:25	Coffee Break
11:25–12:15	Regina Rotman, University of Toronto *Periodic Geodesics and Geodesic Nets on Riemannian Manifolds*
12:20–13:50	Lunch

2. Abstracts of the Plenary Talks

2.1. ***A symplectic approach to systolic inequalities: Spheres of revolution.***

Alberto Abbondandolo, Ruhr Universität Bochum.
E-mail address: alberto.abbondandolo@rub.de.

A mix of classical ideas from dynamical systems and symplectic geometry can be used in order to obtain sharp systolic inequalities in various settings. In this talk, I will present the general principle and focus on the special case of spheres of revolution.
The talk is based on some joint papers with Barney Bramham, Umberto Hryniewicz and Pedro Salomão [1, 2, 3].

References

[1] A. Abbondandolo, B. Bramham, U. L. Hryniewicz, and P. A. S. Salomão, *A systolic inequality for geodesic flows on the two-sphere*, Math. Ann. **367** (2017), 701–753.
[2] ———, *Sharp systolic inequalities for Reeb flows on the three-sphere*, Invent. Math. **211** (2018), 687–778.
[3] ———, *Sharp systolic inequalities for Riemannian and Finsler spheres of revolution*, `arXiv:1808.06995 [math.SG]` (2018).

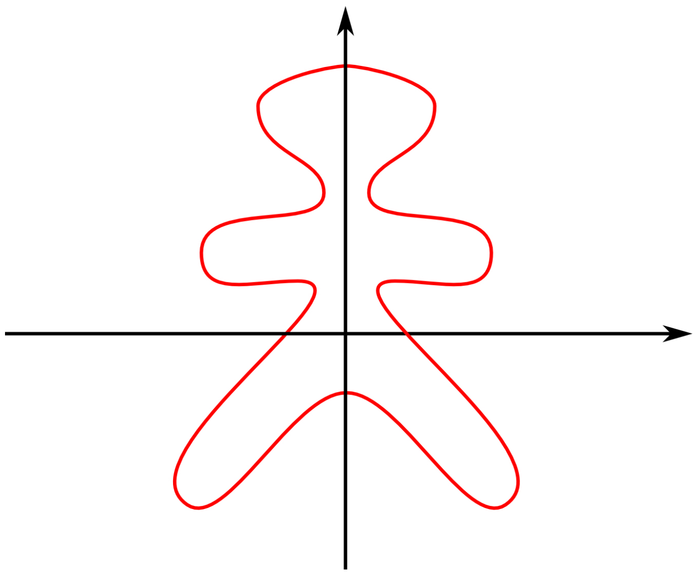

Figure 1. A sphere of revolution (in fieri).

2.2. ***A magnetic systolic inequality on surfaces.***
Gabriele Benedetti, University of Heidelberg.
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Let Σ be a compact oriented surface without boundary endowed with a Riemannian metric g and a function $b : \Sigma \to \mathbb{R}$. A magnetic geodesic is a curve on Σ with geodesic curvature equal to b at every point. We say that (g, b) is Zoll, if all magnetic geodesics are closed with the same minimal magnetic length. Homogeneous Zoll pairs are known to exist on every Σ and, in a work in progress with Luca Asselle, we construct the first non-homogeneous examples on a two-torus. With Jungsoo Kang, we proved in [1] a sharp systolic inequality for pairs (g, b) which are close to a Zoll one: namely, we give an upper bound on the minimal magnetic length of closed magnetic geodesics in terms of a global quantity depending only on the g-volume and genus of Σ and the integral of b over Σ. The upper bound is attained precisely when (g, b) is Zoll.

References

[1] Gabriele Benedetti and Jungsoo Kang, On a systolic inequality for closed magnetic geodesics on surfaces. Preprint on arXiv:1902.01262, 26 pages, 2019.

2.3. ***Intrinsic volumes in contact and pseudo-Riemannian geometries.***
Dmitry Faifman, University of Montréal.
E-mail address: dfaifman@gmail.com.

The intrinsic volumes, or Lipschitz-Killing curvatures, are finitely additive measures on nice subsets of Riemannian manifolds, with roots in convex, differential, and integral geometries. They include the volume, surface area, and Euler characteristic. In recent years, they were considered in the framework of the theory of valuations on manifolds, and many extensions and generalizations of the Riemannian picture started to emerge. We will consider two geometric settings where similar quantities can be defined: pseudo-Riemannian geometry, and contact manifolds. The Weyl principle - that of invariance to isometric embeddings - is a recurring theme. Another is the appearance of Crofton formulas, in spite of a non-compact isotropy group.
Partially based on a joint work in progress with A. Bernig and G. Solanes.

2.4. ***Big mapping class groups acting on homology.***
Federica Fanoni, CNRS and University of Strasbourg.
E-mail address: fanoni@math.unistra.fr.

To try and understand the mapping class group of a surface (informally speaking, its group of symmetries), it is useful to look at its action on the first homology of the surface. For finite-type surfaces this action is fairly well understood. I will discuss joint work with Sebastian Hensel and Nick Vlamis in which we deal with infinite-type surfaces (i.e. whose fundamental group is not finitely generated).

2.5. ***Sublinearly many systoles that fill.***

Maxime Fortier Bourque, University of Glasgow.

E-mail address: maxime.fortier-bourque@glasgow.ac.uk.

I will describe a construction of closed hyperbolic surfaces whose systoles fill, with the number of systoles growing sublinearly in the genus. This seems to indicate that the set of surfaces whose systoles fill, which was proposed by Thurston as a spine for moduli space, has much larger dimension than the virtual cohomological dimension of the mapping class group. These surfaces happen to be critical points of low index for the systole function (a topological Morse function on Teichmüller space), which disproves a conjecture of Schmutz Schaller.

2.6. ***Valuations in Riemannian geometry.***

Joe Fu, University of Georgia.

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A famous theorem of Weyl from 1939 states that, given a smoothly embedded Riemannian manifold M in euclidean space, the volume of a tubular neighborhood of small radius r around M is expressed by a polynomial in r with coefficients given by integrals of certain scalar invariants of the curvature tensor of M. They are usually called Lipschitz-Killing curvatures. It is natural to view the LK curvatures as valuations, i.e. finitely additive functionals on suitably smooth subsets of M, a notion that first arose in convexity theory. We describe how the LK valuations fit into a broader algebraic structure that reveals new geometric phenomena. The story also admits a natural extension to Kaehler manifolds.
These results are due to various subsets of A.Bernig, G. Solanes, T. Wannerer, and the speaker.

2.7. ***Two periodic Reeb orbits.***

Jean Gutt, Universität zu Köln.

E-mail address: gutt@math.uni-koeln.de.

We shall present in this talk a joint result with M. Abreu, J. Kang and L. Macarini on the existence of periodic Reeb orbits. More precisely, that every non-degenerate Reeb flow on a closed contact manifold M admitting a strong symplectic filling W with vanishing first Chern class carries at least two geometrically distinct closed orbits provided that the positive S^1-equivariant symplectic homology of W satisfies a mild condition. We shall then provide numerous example of manifolds satisfying this “mild condition”.

2.8. ***Symplectic methods for sharp systolic inequalities.***

Umberto Hryniewicz, RWTH Aachen.
E-mail address: umbertolh@gmail.com.

In this talk I would like to explain how methods from symplectic geometry can be used to prove a conjecture due to Babenko-Balacheff on the local systolic maximality of the round 2-sphere. Joint work with Abbondandolo, Bramham and Salomao.

2.9. ***Mahler's conjecture for some hyperplane sections.***

Roman Karasev, Moscow Institute of Physics and Technology.
E-mail address: r_n_karasev@mail.ru.

Mahler conjectured in 1939 that for a centrally symmetric convex $K \subset \mathbb{R}^n$ and its polar K° the following inequality holds:

$$vol K \cdot vol K^\circ \geq \frac{4^n}{n!}.$$

There were several results (Bourgain, V. Milman, Nazarov, G. Kuperberg) establishing a weaker inequality, best with $\frac{\pi^n}{n!}$ on the right hand side, a proof of the case $n = 2$ in the original work of Mahler, and a recent preprint by Iriyeh and Shibata, claiming the case $n = 3$.

Despite some results present, among the known approaches the only one that is plausible to give the precise inequality and somehow explain the known not quite obvious equality cases is the reduction of Mahler's conjecture to Viterbo's conjecture on the Ekeland–Hofer–Zehnder capacity and the volume of a convex body $X \subset \mathbb{R}^{2n}$,

$$vol X \geq \frac{c_{EHZ}(X)^n}{n!}.$$

This reduction was made by Artstein-Avidan, Ostrover, and the lecturer in 2014 by establishing the equality

$$c_{EHZ}(K \times K^\circ) = 4$$

for centrally symmetric $K \subset \mathbb{R}^n$.
Although the symplectic approach is not easy to apply, and Viterbo's conjecture is wide open in dimensions $2n \geq 2$, in this talk I will explain an (maybe first substantial) application of symplectic ideas in the very particular case when $K \subset \mathbb{R}^n$ is a hyperplane section of a cube in $\mathbb{R}^{n+1}$, this case does not need the resolution of Viterbo's conjecture and only uses the relatively easy concept of the symplectic reduction.

References

[1] R. Karasev, Mahler's conjecture for some hyperplane sections. Arxiv preprint 1902.08971 (2019).

2.10. **MIN-MAX *characterizations of besse and zoll reeb flows.***
Marco Mazzucchelli, CNRS and École Normale Supérieure de Lyon.
E-mail address: marco.mazzucchelli@ens-lyon.fr.

A closed Riemannian manifold is called Zoll when its unit-speed geodesics are all periodic with the same minimal period. This class of manifolds has been thoroughly studied since the seminal work of Zoll, Bott, Samelson, Berger, and many other authors. It is conjectured that, on certain closed manifolds, a Riemannian metric is Zoll if and only if its unit-speed periodic geodesics all have the same minimal period. In this talk, I will first discuss the proof of this conjecture for the 2-sphere, which builds on the work of Lusternik and Schnirelmann. I will then present a stronger version of this statement valid for general Reeb flows on closed contact 3-manifolds: the closed orbits of any such Reeb flow admit a common period if and only if every orbit of the flow is closed. Time permitting, I will also summarize some related results for Reeb flows on higher dimensional contact spheres and for geodesic flows on simply connected compact rank-one symmetric spaces.
The talk is based on joint works with Suhr, Cristofaro Gardiner and Ginzburg-Gürel.

2.11. ***Filling metric spaces.***
Alexander Nabutovsky, University of Toronto.
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Uryson k-width of a metric space X measures how close X is to being k-dimensional. Several years ago Larry Guth proved that if M is a closed n-dimensional manifold, and the volume of each ball of radius 1 in M does not exceed a certain small constant $\epsilon_n > 0$, then the Uryson $(n-1)$-width of M is less than 1.
Guth conjectured that a much stronger and more general result holds true: For each m there a constant $\epsilon_m > 0$ with the following property: If the m-dimensional Hausdorff content of each metric ball of raidus 1 in a compact metric space X does not exceed ϵ_m, then the $(m-1)$-dimensional Uryson width of X is less than 1.
Such a result leads to interesting new inequalities even for closed Riemannian manifolds. In particular, famous Gromov's systolic inequality for essential manifolds can be simultaneously improved and extended to a much wider class of manifolds.
In my talk I am are going to discuss a joint work with Yevgeny Liokumovich, Boris Lishak and Regina Rotman towards establishing Guth's conjecture.

2.12. ***Isoperimetric Profiles of Surfaces and some related questions in systolic and coarse geometry.***
Panos Papasoglu, Oxford University.
E-mail address: papazoglou@maths.ox.ac.uk.

We will give an example of a surface with a discontinuous isoperimetric profile-answering a question posed by Nardulli-Pansu (joint work with E. Swenson). Our construction uses expander graphs and it turns out similar ideas are relevant in systolic and coarse geometry. We will explain the relationship between these two types of geometry and state some open problems in both areas.

2.13. ***Universal properties of random surfaces.***
Bram Petri, University of Bonn.
E-mail address: bpetri@math.uni-bonn.de.

Random surfaces provide a way to understand the geometry of a "typical" surface. Besides this, they can sometimes be used to prove the existence of surfaces with certain extremal properties.
There are many ways to give meaning to the notion of a random surface. That is, there are many different probability spaces of surfaces that one can define. In this talk, I will speak about geometric and topological properties of random surfaces that are universal among all these models.
This is joint work with Thomas Budzinski and Nicolas Curien.

2.14. ***Periodic Geodesics and Geodesic Nets on Riemannian Manigolds.***
Regina Rotman, University of Toronto.
E-mail address: rina@math.toronto.edu.

I will talk about periodic geodesics, geodesic loops, and geodesic nets on Riemannian manifolds. More specifically, I will discuss some curvature-free upper bounds for compact manifolds and the existence results for non-compact manifolds. In particular, geodesic nets turn out to be useful for proving results about geodesic loops and periodic geodesics.

2.15. ***Affine Quermassintegrals and Minkowski Valuations.***
Franz Schuster, Vienna University of Technology.
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The Blaschke-Santaló and the polar Petty projection inequality are two of the best known and most powerful affine isoperimetric inequalities in convex geometric analysis. In particular, they are significantly stronger than the classical Euclidean Urysohn and *the* isoperimetric inequality, respectively. In 1988, Lutwak conjectured that for convex bodies in $\mathbb{R}^n$ and each $1 \leq i \leq n-1$ an affine isoperimetric inequality holds for the socalled ith affine quermassintegral. The special cases $i = 1$ and $i = n-1$ being the Blaschke-Santaló and the polar Petty projection inequality, respectively.

In this talk, we present new isoperimetric inequalities for Minkowski valuations intertwining rigid motions of degree $i = 1$ and $i = n - 1$. These inequalities not only improve the Urysohn and *the* isoperimetric inequality but interpolate between these classical Euclidean inequalities and the Blaschke-Santaló and the polar Petty projection inequality, respectively. Moreover, among these large families of isoperimetric inequalities, the affine ones turn out to be the strongest inequalities. Finally, we also relate the polar volume of Minkowski valuations of degree $2 \leq i \leq n - 2$ to Lutwak's conjecture on affine quermassintegrals.
Based on joint work with A. Berg, C. Haberl, and G. Hofstätter.

2.16. ***Large genus minimal surfaces in positive Ricci curvature.***
Anna Siffert, Max Planck Institute for Mathematics of Bonn.
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We use Colding–Minicozzi lamination theory to study the systole of large genus minimal surfaces in an ambient three-manifold of positive Ricci curvature.
This is joint work with Henrik Matthiesen.

2.17. ***Counting curves of a given type, revisited.***
Juan Souto, University of Rennes.
E-mail address: jsoutoc@gmail.com.

Mirzakhani wrote two papers studying the asymptotic behaviour of the number of curves of a given type (simple or not) and with length at most L. In this talk I will explain a new independent proof of Mirzakhani's results.
This is joint work with Viveka Erlandsson.

2.18. ***Some inequalities obtained via a curvature flow***
Alina Stancu, Concordia University.
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We consider an $SL(n)$-invariant curvature flow, derived in the sense of [2], whose asymptotic behaviour leads to several geometric inequalities among which a case of the conjectured log-Minkowski inequality, [1].
The interest in log-Minkowski inequality in its full generality is due to its equivalence to a stronger version of Brunn-Minkowski inequality and other implications to existing open problems.

References

[1] K. J. Böröczky, E. Lutwak, D. Yang and G. Zhang, "The log-Brunn-Minkowski inequality", Adv. Math. 231 (2012), 1974â1997.
[2] A. Stancu, "Centro-Affine Invariants for Smooth Convex Bodies", IMRN, Vol. 2012 (2012), 2289 – 2320.

3. Abstracts of the Posters

3.1. ***From curves to current.***

Didac Martinez Granado, Indiana University.
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Geodesic currents are the closure of the space of (not necessarily simple) closed curves on a surface, playing an analog role as measured laminations for simple curves. In 1986, Bonahon introduced geodesic currents and proved that the notion of hyperbolic length for curves extends to currents. Since then, many other functions defined in the space of curves have been proven to extend to currents, such as length for negatively curved metrics on a surface (Otal, 90), flat metrics coming from quadratic differentials (Duchin-Leininger-Rafi, 10), stable lengths for surfaces (Erlandsson-Parlier-Souto, 16). In this poster, we explain how a function defined on the space of curves satisfying some simple conditions can be extended continuously to geodesic currents. The most important of these conditions, perhaps, is that the function decreases under smoothing of essential crossings. Our extension result subsumes previous extension results of functions from curves on surfaces to geodesic currents, and extends functions that had not been considered before, such as the extremal length.
This is joint work with Dylan Thurston.

3.2. ***Blaschke-Santaló inequalities for Minkowski Endomorphisms.***

Georg Hofstätter, Technische Universität Wien.
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In this work, we present a family of new isoperimetric inequalities for monotone Minkowski endomorphisms that interpolates between the Blaschke-Santaló and the Urysohn inequality. Among this large family of inequalities, the only affine invariant inequality turns out to be the strongest one. An extension of the family to merely weakly monotone Minkowski endomorphisms is shown to be impossible.
This is a joint work with F.E. Schuster.

3.3. ***Sharp Sobolev Inequalities via Projection Averages.***

Philipp Kniefacz, Vienna University of Technology.
E-mail address: philipp.kniefacz@tuwien.ac.at.

In this work we present a family of sharp Sobolev-type inequalities obtained from averages of the length of i-dimensional projections of the gradient of a function. This family has both the classical Sobolev inequality (for $i = n$) and the affine Sobolev-Zhang inequality (for $i = 1$) as special cases as well as a recently obtained Sobolev inequality of Haberl and Schuster (for $i = n - 1$). Moreover, we identify the strongest member in our family of analytic inequalities which turns out to be the only affine invariant one among them.
This is a joint work with F.E. Schuster.

3.4. ***Ideal polytopal surfaces in Fuchsian manifolds.***
Roman Prosanov, University of Fribourg.
E-mail address: rprosanov@mail.ru.

In 1942 P. Alexandrov proved that every Euclidean metric on the 2-sphere with conical singularities of positive curvature can be uniquely realized (up to isometry) as the induced metric on the boundary of a convex 3-dimensional polytope. It provided a complete inner description of such metrics and was used in the development of a general theory of metrics with nonnegative curvature.
Various authors gave several generalizations of this result. In particular, Jean-Marc Schlenker proved a similar statement about hyperbolic cusp-metrics on surfaces of genus > 1 (realized in Fuchsian manifolds). Another proof was obtained by François Fillastre. Both of them used the non-constructive "deformation method".
We introduce the discrete curvature functional for polytopal manifolds and use it to give a new variational proof in the ideal Fuchsian case.
We also discuss the relation with discrete uniformization theory developed very recently by Gu et al.

3.5. ***Mahler's conjecture for some hyperplane sections.***
Thomas Hack, Technische Universität Wien.
E-mail address: thomas.hack@tuwien.ac.at.

As a natural analog of Urysohn's inequality in Euclidean space, Gao, Hug, and Schneider showed in 2003 that in spherical or hyperbolic space, the probability that a given convex body K meets a randomly chosen totally geodesic hypersurface is minimized when K is a geodesic ball.
We present a random extension of this result by taking K to be the convex hull of finitely many points drawn according to a probability distribution and by showing that the minimum is attained at uniform distributions on geodesic balls.
As a corollary, we obtain a randomized Blaschke–Santaló inequality on the sphere.

4. List of Participants

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