

Combinatorics of decomposition spaces (2-Segal spaces)

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Dyckerhoff's IRTATCA 2015 lectures explained the role of 2-Segal spaces in the theory of Hall algebras; the present follow-up lectures will focus on their role in combinatorics in connection with keywords such as incidence (co)algebras, Möbius inversion and antipodes. From this viewpoint, 2-Segal spaces are seen as an expression of the ability to decompose, just as 1-Segal spaces (categories) express the ability to compose, and for this reason 2-Segal spaces are also called decomposition spaces. One feature of the theory is that it seeks to do "objective combinatorics", which means working with the combinatorial objects themselves rather than with vector spaces spanned by their isomorphism classes. A key ingredient is "*homotopy linear algebra*", where elementary linear algebra is performed with ∞ -groupoids, slice categories, and spans, instead of numbers, vector spaces, and matrices, so that combinatorial identities can be realised as the (homotopy) cardinality of equivalences of (∞ -)groupoids. The lectures will start with classical Möbius inversion in number theory, and gradually introduce more abstract machinery and homotopy viewpoints. (No prior knowledge of ∞ -categories is assumed. They enter only in a rather secondary way, and it is meaningful to substitute the word "*set*" for the word " *∞ -groupoid*".)