16th School on Interactions between Dynamical Systems and Partial Differential Equations

Centre de Recerca Matemàtica

June 25th to 29th, 2018

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2. Abstracts of the Lecturers

Singularity formation and bubbling in nonlinear diffusions
Manuel del Pino, Universidad de Chile in Santiago
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A fundamental question in nonlinear evolution equations is the analysis of solutions which develop singularities (blow-up) in finite time or as time goes to infinity. We review recent results on the construction of solutions to certain notable nonlinear parabolic PDE which exhibit this kind of behavior in the form of "bubbling". This means solutions that at main order look like asymptotically singular time-dependent scalings of a fixed finite energy entire steady state. In these lectures we carry out this analysis forsome specific problems, including the classical two-dimensional harmonic map flow into the sphere and the energy-critical semilinear heat equation.

An introduction to geometric measure theory
Francesco Maggi, The University of Texas at Austin
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The scope of these lectures is introducing students to various key ideas from Geometric Measure Theory (GMT). We will examine the most relevant features of rectifiability, and of the theories of sets of finite perimeter and rectifiable varifolds which have been built upon it. We will explain what is a "small excess regularity criterion", an idea that originated in GMT and that is now fundamental in the analysis of many nonlinear PDE. Time permitting, we will illustrate these ideas in action by showing the existence of minimizers in various formulations of Plateau's problem (following our joint paper with Camillo De Lellis and Francesco Ghiraldin.)

Periodic orbits and invariant tori in ideal fluid flows
Daniel Peralta-Salas, ICMAT Madrid
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Abstract: Ideal fluid flows are modelled by the Euler equations, a complicated system of non-linear PDEs where the unknowns are a non-autonomous vector field representing the velocity of the fluid and a time-dependent scalar function representing the pressure.

The integral curves of the velocity field are the fluid particle paths, so the analysis of these orbits from the dynamical systems viewpoint is relevant to understand the fluid motion. The goal of this course is to present some results on the trajectories of autonomous vector fields that are time-independent (or stationary) solutions of the 3D Euler equations. This (huge) family of vector fields is known as Euler fields.

The course consists of four lectures. In Lecture 1 I will review Arnold's structure theorem on the integrability of typical Euler vector fields. In the second Lecture I will show that any analytic and non-vanishing Euler field on the 3-dimensional sphere has a periodic orbit. The last two lectures will be devoted to the study of a particularly relevant class of Euler fields, known as Beltrami fields, which are eigenfunctions of the curl operator. I will introduce Arnold's conjectures in this setting, and will present two realization theorems for Beltrami flows in Euclidean space. The first theorem shows that, for any locally finite union of pairwise disjoint (possibly knotted and linked) closed curves, there exists a Beltrami field with a set of hyperbolic periodic orbits diffeotopic to this set of curves. The second theorem claims that, for any set of finitely many pairwise disjoint (possibly knotted and linked) embedded tori, there exists a Beltrami field with a set of invariant tori diffeotopic to the aforementioned set.

Time permitting, I will discuss some applications of these results to analyze the existence of bifurcations in time-dependent solutions to the 3D Navier-Stokes equations.

KAM theory for non linear partial differential equations on the circle

Michela Procesi, Università Roma Tre, Italy
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In this course we shall mainly focus on the study of nonlinear Schrodinger type equations on the circle, with the purpose of proving existence and stability of invariant tori which carry a linear flow.

The persitence of finite dimensional invariant tori (quasi-periodic solutions) for semi-linear PDEs on the circle has been successfully studied in the last thirty years and recently there have been some important developments in the fully non-linear case. However there are still interesting open questions, for instance what happens for PDE's on more general compact domains, or —even in the simplest setting— whether infinite dimensional invariant tori persist. We shall give an overview of the classical KAM scheme in a framework which is flexible and suitable for the study of the aforementioned problems, which we shall then discuss.

3. Abstracts of the Posters

Decay estimates in evolution equations with fractional derivatives
Elisa Affili, Università degli Studi di Milano
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We consider an evolution equation with possibly fractional diffusion. The anomalous diffusion may occur either in space or in time (or both) and the diffusion operator can be also nonlinear. We prove some power-law decay estimates in time, finding a polynomial decay that depends on the parameter of the fractional time derivative. We are able to prove the decay by using a structural hypothesis, but many operators do satisfy it. Among these there are the Laplacian, the p-Laplacian, the fractional Laplacian, the porous medium equations and the fractional magnetic operator. In fact, the hypothesis holds essentially when it is possible to perform an integration by parts of the energy functional. We believe that the result is very general and applies to a wide cluster of operators.

The Taylor series invariant of semi-toric systems
Jaume Alonso, University of Antwerp
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Semi-toric systems are a specific class of completely integrable systems in four dimensions where one of the first integrals is proper and induces a global $\mathbb{S}^1$-action, together with some other assumptions. Á. Pelayo and S. Vũ Ngọc presented some years ago a classification of semi-toric systems in terms of five symplectic invariants. One of these invariants, the so-called *Taylor series invariant*, consists of the association of a Taylor series to each of the singularities of the semi-toric system that are of focus-focus type. We present the recent explicit calculation of this invariant for two systems, the coupled spin-oscillator and the coupled angular momenta. The latter depends on some parameters, which can make the singularity of focus-focus type turn into a singularity of elliptic-elliptic type. We also show how the Taylor series invariant reflects this dependence.

This work is joint with H. Dullin (University of Sydney) and S. Hohloch (University of Antwerp).

Solidification at the nanoscale. The Guyer-Krumhansl-Stefan problem [1]
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Advances in manufacturing processes have brought us to the stage where reliable nanoscale devices are now commonplace. However, in most current and predicted applications of nanostructures, there is a strong concern over the management of heat [2]. In addition, nanoscale solidification is becoming increasingly relevant in applications involving ultra-fast freezing processes and nanotechnology. The ability to successfully manipulate heat is therefore vital to device performance and a lack of thermal regulation can lead to melting and device failure [3]. As shown by experimental [4] and theoretical studies [5] thermal transport on the nanoscale is driven by infrequent collisions between thermal energy carriers and is not well described by the classical Fourier law.

In this study, the role of non-Fourier heat conduction is studied by coupling the Stefan condition to the Guyer-Krumhansl equation [6], which is an extension of Fourier's law that includes memory and non-local effects. The results from this study provide key qualitative insights that can be used to control nanoscale solidification processes.

This is a work is joint with Matthew G. Hennessy and Timothy G. Myers.

Acknowledgments: Marc Calvo-Schwarzwälder acknowledges the financial support of the 'la Caixa' Foundation.

References

[1] M.G. Hennessy, M. Calvo-Schwarzwälder, and T.G. Myers, *Asymptotic analysis of the Guyer-Krumhansl-Stefan model for nanoscale solidification.* Submitted to Applied Mathematical Modelling, November 2017.

[2] D.G. Cahill, W.K. Ford, K.E. Goodson, G.D. Mahan, A. Majumdar, H.J. Maris, R. Merlin, and S.R. Phillpot, *Nanoscale thermal transport.* Journal of Applied Physics, 93 (2003), no. 2, 793–818.

[3] A. Nie, J. Liu, C. Dong, and H. Wang, *Electrical failure behaviours of semiconductor oxide nanowires.* Nanotechnology, 22 (2011), no. 40, 405703.

[4] C.-W. Chang, D. Okawa, H. Garcia, A. Majumdar, and A. Zettl, *Breakdown of Fourier's law in nanotube thermal conductors.* Physical Review Letters, 101 (2008), no. 7, 075903.

[5] D. Jou, J. Casas-Vázquez, and G. Lebon, Extended irreversible thermodynamics. Springer, 1996.

[6] R.A. Guyer and J.A. Krumhansl, Solution of the linearized phonon Boltzmann equation. Physical Review 148 (1966), 766.

Critical energy in soliton-defect interaction models

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We present a toy-model of interaction of kinks of the sine-Gordon equation with a weak defect. In general, one can understand the kink-defect interaction as a small perturbation (modeled by a Dirac delta function) of a travelling wave.

We consider a finite-dimensional reduction of the equation given by a 2-degrees of freedom Hamiltonian H, and we propose a geometric approach to give conditions on the energy of the system to admit kinks. More precisely, we obtain an asymptotic expression for the critical energy h_c such that the system admits kinks with small amplitude only for $h \geqslant h_c$.

Roughly speaking, our methods rely on computing the exponentially small transversality of invariant manifolds $W^{u,s}$ of certain objects (critical points and periodic orbits) at infinity.

This is a joint work with M. Guardia and T.M. Seara.

Persistence of normally hyperbolic invariant manifolds in the absence of rate conditions

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We consider perturbations of normally hyperbolic invariant manifolds, under which they can loose their hyperbolic properties. We show that if the perturbed map which drives the dynamical system preserves the properties of topological expansion and contraction, then the manifold is perturbed to an invariant set. The main feature is that our results do not require the rate conditions to hold after the perturbation. In this case the manifold can be perturbed to an invariant set, which is not even a topological manifold. Our method is not perturbative. It can be applied to establish existence invariant sets within a prescribed neighborhood also in the absence of a normally hyperbolic invariant manifold prior to perturbation. We work is in the setting of nonorientable Banach vector bundles, without needing to assume invertibility of the map.

This is a joint work with Maciej Capiński.

Machine-learning prediction of energy functions from data using reservoir computation

Kengo Nakai, University of Tokyo
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We predict energy functions of fluid flow using reservoir systems constructed from data. In our procedure of the prediction, we assume no prior knowledge of physical model describing fluid flow except that its behavior is complex but deterministic. We show that the reservoir dynamics constructed from only past data of energy functions for every wavenumber can predict the behavior of energy functions and reproduce the energy spectrum. This implies that the obtained reservoir system constructed without the knowledge of microscopic data is equivalent to the dynamical system describing macroscopic behavior of energy functions.

Splitting and rigidity theorems for the nonlinear Poisson equation on complete Riemannian manifolds

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We review the pointwise gradient upper bound of bounded solutions of the Poisson equation and generalize it to a wider geometric setting. The study of the achievement of this bound leads to a splitting theorem.

This is a work is joint with A. Farina (Université de Picardie Jules Verne).

Singularities of minimum time control of mechanical systems
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This poster will focus on recent developments on optimal time control of mechanical systems, with in mind application to the controlled Kepler and circular restricted three body-problems (CRTBP). We are interested in minimizing the final time for affine control systems,

$$\text{(1)} \quad \begin{cases} \dot{x}(t) = F_0(x(t)) + u_1(t)F_1(x(t)) + u_2(t)F_2(x(t)), \quad t \in [0, t_f], \quad u \in B, \\ x(0) = x_0, \quad x(t_f) = x_f, \\ t_f \to \min, \end{cases}$$

where the control u is contained in the euclidean ball B, the F_i are smooth vector fields, and the phase space M is a four dimensional manifold. (Most results are valid of $2m$ dimensional manifold with m controls.) Pontrjagin Maximum Principle provides the following necessary condition: Optimal trajectories are projections on M of so-called extremals, that is of solutions of the Hamiltonian system defined on T^*M by

$$H^*(z) = H_0(z) + \sqrt{H_1^2(z) + H_2^2(z)},\ z = (x, p) \in T^*M$$

with $H_i(x, p) = \langle p, F_i(x)\rangle$, $i = 0, 1, 2$. The associated control is

$$u = \frac{1}{\sqrt{H_1^2 + H_2^2}}(H_1, H_2).$$

The set $\Sigma = \{z \in T^*M,\ H_1(z) = H_2(z) = 0\}$ defines a singular locus, and we are interested in the behaviour of the Hamiltonian flow in the neighborhood of Σ. When this set is crossed, the control admits a discontinuity, called a switch. We partition Σ into three subsets Σ_-, Σ_+, and Σ_0, on which we study the flow. We make the following generic assumption:

$$\text{(A)} \qquad \det(F_1(x), F_2(x), F_{01}(x), F_{02}(x)) \neq 0,\ \text{for almost all } x \in M,$$

This assumption is in particular valid for every second order controlled mechanical system of the form (V denotes a potential)

$$\text{(2)} \qquad \ddot{q} + \nabla V(q) = u.$$

We use a blow up and give a normal form for the extremal system, which allows us to prove:

Theorem. *Assume (A) holds, then there is a unique extremal (with a switch) passing through each* $\bar{z} \in \Sigma_-$*; the extremal flow is locally well defined and there exist a stratification of the phase space such that the flow is smooth on each stratum. Furthermore, when crossing the strata, the flow admits log-type singularities (and thus belongs to the log-exp category).*

In a tubular neighborhood of Σ_+, the flow is smooth and no extremal crosses Σ_+ (there is no switch, thus). In the last case (Σ_0), we prove the following:

Proposition 1. *For every point $\bar{z} \in \Sigma_0$, there exists an extremal having a switch at $\bar{z}$.*

In the case of mechanical system like (2), the vector fields F_1 and F_2 commute, and the switches are instantaneous rotations of angle π of the control (the so-called π-singularities, see [3]). We can obtain a global bound on the number of such singularities in the Kepler and CRTBP cases.

Finally, we would like to present the following result in the Kepler case:

Theorem. *The minimum time Kepler problem defines a non-integrable Hamiltonian system (in the Liouville sense).*

This is a joint work with J.-B. Caillau (Nice), J. Féjoz (Paris) and R. Roussarie (Dijon).

References

[1] J.-B. Caillau, T. Combot, J. Féjoz, and M. Orieux, Non-integrability of the minimum time Kepler problem. In revision for J. Geom. Phys. (2018)

[2] J.-B. Caillau, J. Féjoz, M. Orieux, and R. Roussarie, On singularities of min time control affine systems. Preprint (2018)

[3] J.-B. Caillau and B. Daoud, Minimum time control of the restricted three-body problem. SIAM J. Control Optim. 50 (2012), no. 6, 3178–3202.

On solvability of fractional models of viscoelastic fluid

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The mathematical models of dynamics of viscoelastic fluids with constitutive relations containing fractional derivatives are under consideration. Fractional analogous of Voigt and anti-Zener models are under investigation. We establish the existence of weak solutions of the corresponding initial-boundary value problems. In the planar case the uniqueness of weak solutions is proved. For the proofs of the main results we approximate the problems under consideration by a sequence of regularized systems of Navier-Stokes type. The solvability of regularized systems and apriori estimates of their solutions allow to pass to the limit in the regularized systems and obtain the solvability of original problems. The theory of fractional powers of positive operators, fractional calculus and classical results on Navier-Stokes equations are used.

This is a joint work with Victor Zvyagin.

This research was supported by the Ministry of Education and Science of the Russian Federation (grant 14.Z50.31.0037).

References

[1] V.G. Zvyagin and V.P. Orlov, On the weak solvability of the Problem of viscoelasticity with memory. Differential equations 53 (2017), no. 2, 215–220.
[2] V.G. Zvyagin and V.P. Orlov, On problem of the dynamics of a viscoelastic medium with memory on an infinite interval. Doklady Mathematics 96 (2017), no. 1, 329–331.
[3] V.G. Zvyagin and V.P. Orlov, Solvability of a parabolic problem with non smooth data. Journal of Mathematical Analysis and Applications 453 (2017), no. 1, 589–606.
[4] V.G. Zvyagin and V.P. Orlov, Weak solvability of a fractional Voigt viscoelasticity model. Doklady Mathematics 96 (2017), no. 2, 1–3.

A quantitative version of Alexandrov's Soap Bubble Theorem

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et M be the Euclidean space or the hyperbolic space. The celebrated Alexandrov's Soap Bubble Theorem states that spheres are the only closed (i.e. compact and without boundary) constant mean curvature hypersurfaces embedded in M. The poster maily focuses on the following quantitative version of the Alexandrov Theorem

Theorem (Ciraolo-Vezzoni [1, 2]). *Let S be an m-dimensional, C^2-regular, connected, closed hypersurface embedded in M. There exist constants ε, $C > 0$ such that if*

$$\operatorname{osc}(H) \leqslant \varepsilon,$$

then there are two concentric balls B_{r_i} and B_{r_e} such that

$$S \subset B_{r_e} \backslash \bar{B}_{r_i}$$

and

$$r_e - r_i \leqslant C \operatorname{osc}(H).$$

The constants ε and C depend only on m and upper bounds on the C^2-regularity and the area of S.

The proof of the Theorem makes use of a quantitative study of the method of the moving planes and the result implies a new pinching Theorem for hypersurfaces. Furthermore, the Theorem is optimal in a sense that will be specified in the poster. As far as the organization of the poster: the formal statement of the cited Theorems, the original proof of Alexandrov with the moving planes method and the quantitative studies of this method will be presented. There will be also a section about an on-going study, in collaboration with G. Ciraolo and L. Vezzoni, on the generalization of the result.

References

[1] G. Ciraolo and L. Vezzoni., A sharp quantitative version of Alexandrov's theorem via the method of moving planes. J. Eur. Math. Soc. (JEMS), 20 (2018), 261–299.
[2] G. Ciraolo and L. Vezzoni, Quantitative stability for hypersurfaces with almost constant mean curvature in the hyperbolic space. Preprint `ArXiv:1611.02095`.

Attractors for the Bingham model in three-dimensional case

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The Bingham model of media motion is often used to describe the motion of viscoplastic media. It was investigated from the point of view of the existence theorems of solutions in [1]. The attractors of this model in the two-dimensional case were studied in [2] using the classical theory of dynamical systems, for which the uniqueness of solutions is a necessary condition. However, for the considered system of equations in the three-dimensional case the uniqueness of weak solution not known. For an autonomous situation, we prove the existence of a trajectory and global attractors in the three-dimensional case under periodic conditions with respect to spatial variables. To prove the existence of attractors, an abstract theory of trajectory and global attractors was used (see [3]), in which uniqueness is not required.

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Periodic solutions of state-dependent impulsive differential equations

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Many evolution processes are characterized by the fact that they experience a sudden change in their state at certain moments of time. It is natural to assume that these changes act instantaneously and in the form of impulses. Mathematical models in aircraft control, population dynamics, economics or drug administration show impulses effects. The moments of impulse effects could be fixed beforehand or they could depend on the solution.

We study a second-order differential equation with state-dependent impulses. The existence of periodic solutions is obtained by means of a new twist fixed-point theorem, similar to Poincaré's last geometric theorem.

Solvability of thermoviscoelastic model of polymer motion

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We consider the following initial boundary value problem

$$\frac{\partial v}{\partial t}+\sum_{i=1}^{n} v_i\frac{\partial v}{\partial x_i}-2\mathrm{Div}(\nu(\theta)\mathcal{E}(v))-\varkappa\frac{\partial \Delta v}{\partial t}-2\varkappa\mathrm{Div}\big(v_i\frac{\partial \mathcal{E}(v)}{\partial x_i}\big)+\nabla p=f,\ (t,x)\in Q_T;$$

$$\mathrm{div}\ v=0,\quad (t,x)\in Q_T;\qquad v|_{t=0}=v_0,\quad x\in\Omega;\qquad v|_{[0,T]\times\partial\Omega}=0;$$

$$\frac{\partial \theta}{\partial t}+\sum_{i=1}^{n} v_i\frac{\partial \theta}{\partial x_i}-\chi\Delta\,\theta=2\nu(\theta)\mathcal{E}(v):\mathcal{E}(v)+2\varkappa\frac{\partial \mathcal{E}(v)}{\partial t}:\mathcal{E}(v)+g,\quad (t,x)\in Q_T;$$

$$\theta|_{t=0}=\theta_0,\quad x\in\Omega;\qquad \theta|_{[0,T]\times\partial\Omega}=0.$$

Here $\Omega \subset \mathbb{R}^n$, $n = 2,3$, $v(x,t) = (v_1,...,v_n)$ is the velocity vector-function, $\theta(t,x)$ is the temperature function, $p(x,t)$ is the fluid pressure, $f(x,t)$ is the density of external forces, g is the external heat source, $\mathcal{E}(v) = (\mathcal{E}_{ij}(v))_{j=1,...,n}^{i=1,...,n}$, $\mathcal{E}_{ij}(v) = \dfrac{1}{2}(\dfrac{\partial v_i}{\partial x_j} + \dfrac{\partial v_j}{\partial x_i})$ is the strain velocity tensor, $\chi > 0$ is the coefficient of thermal conductivity, $\varkappa > 0$ is the time of retardation (delay), $\mu_0 > 0$ is the initial viscosity of a fluid, $\mu(s)$ is the viscosity of a fluid and $\widetilde{\mu}(s) = \mu_0(s) + \mu(s)$.

The initial-boundary value problem under consideration describes the weakly concentrated water polymer solutions motion. This problem is considered with constitutive law which is frame indifferent, i.e. that do not change under the Galilean transformation. Also in this mathematical model the viscosity depends on a temperature, which leads to emergence of additional heat balance equation (it is a parabolic equation with nonsmooth coefficients and with right part from $L_1(0,T;L_1(\Omega))$. For the initial-boundary value problem under consideration the existence theorem of weak solutions is proved. For this the topological approximation approach for hydrodynamic problems is used.

Acknowledgments This research was supported by the Ministry of Education and Science of the Russian Federation (grant 14.Z50.31.0037).

References

[1] A.V. Zvyagin, Solvability for equations of motion of weak aqueous polymer solutions with objective derivative. Nonlinear Analysis: Theory, Methods and Applications 90 (2013), 70–85.
[2] A.V. Zvyagin and V.P. Orlov, Solvability of the thermoviscoelasticity problem for linearly elastically retarded Voiht liquids. Mathematical Notes, 97 (2015), no. 5, 38–52.

Attractors of viscoelastic Jeffreys-Oldroyd-Lethersich model

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The following initial boundary value problem is considered

$$\frac{\partial v}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial v}{\partial x_i} - \mu_1 \Delta v + \text{grad } p = \text{Div } \tau + f, \tag{3}$$

$$\tau + \lambda_1 (\frac{\partial \tau}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial \tau}{\partial x_i}) = 2(\eta - \mu_1)\mathcal{E}, \tag{4}$$

$$\text{div } v = 0, v\big|_{\partial\Omega} = 0. \tag{5}$$

Here Ω is an arbitrary bounded domain in $\mathbb{R}^n$ $(n = 2,3)$, v is an unknown vector-function of velocity, p is an unknown function of pressure, τ is a non-Newtonian component of the shear stress tensor. All of them depend on a point $x \in \Omega$ and on a moment of time t; $f(x)$ is a density of external forces, $\mathcal{E} = (\mathcal{E}(v)_{ij})$, $\mathcal{E}_{ij} = \frac{1}{2}(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})$ is the strain velocity tensor, $\eta > 0$ is the viscosity of the medium, $\lambda_1 > 0$ is the relaxation time, $\mu_1 < \eta$ is a positive constant.

This system corresponds to the Jeffreys-Oldroyd-Lethersich model. It describes in various situations the motion of the following viscoelastic medias: bitumens, polymers and their solutions, blood, dough, earth crust, concrete and etc. (see [1]–[3]).

At the study of the systems dynamics the limit behavior as the time tends to infinity is of special interest. In practice, systems with the following property often occur: "at infinity" their dynamics is concentrated on a small part of the phase space called an attractor. Whatever the initial state of the system is "forgotten" with time, and the state of the system approaches the attractor arbitrarily closely. In the case of such systems, it is natural to study the dynamics precisely on attractors, since the states that do not belong to attractors are certainly "transient". This is why studying the existence and properties of attractors is topical for mathematical problems in modern science and, in particular, in fluid dynamics.

Attractors for models of Newtonian hydrodynamics is well known (O.A. Ladyzhenskaya, M.I. Vishik, etc.). But models of non-Newtonian hydrodynamics require the development of a more general abstract theory (see [3]). On the basis of this developed attractors theory attractors of the system (1)-(3) are considered.

It will be considered the existence of

1) attractors of the system (1)–(3) in the autonomous case,
2) uniform attractors of the system (1)–(3) in the non-autonomous case,
3) pullback-attractors of the system (1)-(3) in non-autonomous case.

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References

[1] M. Reiner, Reology. In Handbuch der Physik, S. Flugge (Ed.), Bd. VI, Springer, 1958.

[2] J.G. Oldroyd, Non-Newtonian ow of liquids and solids. Rheology: Theory and Applications, Vol. I (F.R. Eirich, Ed.), AP, New York (1956), 653–682.

[3] V.G. Zvyagin and D.A. Vorotnikov, Topological approximation methods for evolutionary problems of nonlinear hydrodynamics. Walter de Gruyter, Berlin-New York, 2008.

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