Finite Dimensional Integrable Systems in Geometry and Mathematical Physics (FDIS 2017)

Centre de Recerca Matemàtica

July 3rd to 7th, 2017

Organizing Committee

Amadeu Delshams, Universitat Politècnica de Catalunya
Yuri Fedorov, Universitat Politècnica de Catalunya
Vladimir S. Matveev, Universität Jena
Eva Miranda, Universitat Politècnica de Catalunya
Francisco Presas, CSIC–ICMAT Madrid
Sergei Tabachnikov, Pennsylvania State University

Scientific Committee

Alexey Bolsinov, Loughborough University
Allan P. Fordy, University of Leeds
Boris Kruglikov, University of Tromsø
Andrzej Maciejewski, University of Zelena Gora
Juan Jose Morales Ruiz, Universidad Politécnica de Madrid
Emma Previato, Boston University
Vladimir Rubtsov, University of Angers
Dmitry Treschev, Steklov Mathematical Institute
Galliano Valent, Université Paris VI, LPTHE
Nguyen Tien Zung, Université Toulouse III

Key Speakers

Anton Alekseev, University of Geneva
Michael Bialy, Tel Aviv University
Alexey Bolsinov, Loughborough University
Sergei Gukov, California Institute of Technology
Vadim Kaloshin, University of Maryland
Andreas Knauf, Friedrich-Alexander-Universität Erlangen
David Martínez Torres, Pontificia Universidade Catolica do Rio de Janeiro
Emma Previato, Boston University
Maria Przybylska, University of Zielona Gora
Yuri B. Suris, Technische Universität Berlin
Dmitry Treschev, Moscow State University
Pol Vanhaecke, Université de Poitiers
Jonathan Weitsman, Northeastern University

Acknowledgements: The *Finite Dimensional Integrable Systems in Geometry and Mathematical Physics* is made possible in part by the generous support from the Alfred P. Sloan Foundation, the National Science Foundation, and UPC.

Contents

1. Schedule

Monday, 3rd of July 2017	
09:00 – 09:25	Registration
09:25 – 09:30	Opening
09:30 – 10:20	Dmitry Treschev, Moscow State University *Integrability and chaos: coexistence in one system*
10:30 – 11:20	Vadim Kaloshin, University of Maryland *Birkhoff Conjecture for convex planar billiards*
11:20 – 11:50	Coffee break
11:50 – 12:40	Sergei Gukov, California Institute of Technology *Equivariant quantization of the Hitchin moduli space*
12:50 – 13:25	Božidar Jovanović, University of Belgrade *Billiards on constant curvature spaces*
13:30 – 14:00	Oleg Chalykh, University of Leeds *Dunkl–Cherednik operators and quantum Lax pairs*
14:00 – 15:00	Lunch
15:00 – 15:30	Elena Kudryavtseva, Moscow State University *Continuous orbital invariants of integrable Hamiltonian systems*
15:40 – 16:30	David Martínez Torres, Pontificia Universidade Catolica do Rio de Janeiro *Desingularizations of Poisson structures of compact type and action-angle coordinates*
16:40 – 17:30	Maria Przybylska, University of Zielona Gora *Dynamics of constrained many body problems on sphere and rigid bodies with movable l points*
17:30	Welcome Wine and Cheese and Poster Session

Tuesday, 4th of July 2017	
09:30 – 10:20	Alexey Bolsinov, Loughborough University *Bochner-flat (pseudo-)Kähler metrics, Manakov-type operators and new symmetric spaces* *TBA*
10:30 – 11:20	Michael Bialy, Tel Aviv University *Algebraic non-integrability of billiards*
11:20 – 11:50	Coffee break
11:50 – 12:40	Andriy Panasyuk, University of Warmia and Mazury *On local bisymplectic realizations of compatible Poisson brackets*
12:50 – 13:25	Boris Kruglikov, University of Tromsø *Local non-existence of polynomial integrals for geodesic flows*
13:30 – 14:00	Witold Respondek, Normandie Université *Integrability properties of the geodesic equation in sub-Riemannian*
14:00 – 15:00	Lunch
15:00 – 18:00	Parallel sessions

Afternoon parallel sessions	
Singularities and bifurcations diagrams	
15:00 – 15:25	Andrey Oshemkov, Lomonosov Moscow State University *Bifurcation diagram for Adler – van Moerbeke integrable case*
15:35 – 16:00	Joseph Palmer, Rutgers, the State University of New Jersey *New constructions of semitoric integrable systems*
16:00 – 16:25	Nikolay Martynchuk, University of Groningen *Knauf's topological degree and scattering monodromy*
16:25 – 17:00	Coffee break
17:00 – 17:25	Anton Izosimov, University of Toronto *On smooth almost direct product decompositions of non-degenerate*
17:35 – 18:00	Vladislav Kibkalo, Moscow State University *Bifurcations of the Liouville foliations for the Kovalevskaya case*
Killing tensors	
15:00 – 15:25	Gianni Manno, Politecnico di Torino *Normal forms of 2-dimensional metrics with projective vector fields I*
15:35 – 16:00	Andreas Vollmer, INdAM - Politecnico di Torino *Normal forms of 2-dimensional metrics with projective vector fields II*
16:00 – 16:25	Andrei Konyaev, Moscow State University *Singular points of Nijenhuis operator and left-symmetric algebras*
16:25 – 17:00	Coffee break
17:00 – 17:25	Daisuke Tarama, Ritsumeikan University *Integrability of geodesic flow on nilpotent Lie groups of pseudo-H-type*
17:35 – 18:00	Kentaro Tomoda, Kobe University *On integrability of the Killing equation*

Wednesday, 5th of July 2017	
09:30 – 10:20	Emma Previato, Boston University *Of relationships between the Toda chain, QRT maps, Poncelet's porism and the Painlevé VI equation*
10:30 – 11:20	Pol Vanhaecke, Université de Poitiers *On the geometric quantization of (some) Poisson manifolds*
11:20 – 11:50	Coffee break
11:50 – 12:40	Yuri B. Suris, Technische Universität Berlin *Pluri-Lagrangian systems, Sklyanin's spectrality property, and billiards in confocal quadrics*
12:50 – 13:25	Allan P. Fordy, University of Leeds *Poisson algebras and 3D superintegrable Hamiltonian systems*
13:30 – 14:00	Harry Braden, University of Edinburgh *On the construction of monopoles*
14:00 – 15:00	Lunch
17:00	Guided tour to the Gothic quarter in Barcelona (registration is needed)
20:30	Dinner at Balthazar Restaurant (registration is needed)

Thursday, 6th of July 2017	
09:30 – 09:55	László Fehér, University of Szeged and Wigner RCP *Integrable many-body models in action-angle duality from reductions of Heisenberg doubles*
10:00 – 10:25	Angel Ballesteros, University of Burgos *Integrable Hénon–Heiles systems on curved spaces*
10:30 – 10:55	Sonja Hohloch, Universiteit Antwerpen *On recent progress in semitoric systems. Taylor series invariant for coupled angular momenta and coupled spin oscillators, a new system with 2 focus-focus points, and vertical almost toric systems*
10:55 – 11:20	Romero Solha, PUC-Rio *Geometric quantisation of semitoric integrable systems and almost toric manifolds*
11:20 – 11:50	Coffee break
11:50 – 12:40	Andreas Knauf, Friedrich-Alexander-Universität Erlangen *Celestial mechanics: Aspects of integrability*
12:50 – 13:25	Sergey Bolotin, Moscow Steklov Mathematical Institute and University of Wisconsin-Madison *Topology, singularities and integrability in Hamiltonian systems with two degrees of freedom*
13:30 – 14:00	Charles Jaffé, West Virginia University *The role of homographic dynamics in atomic and molecular systems*
14:00 – 15:00	Lunch
15:00 – 18:00	Parallel sessions

Afternoon parallel sessions	
Lax pairs, separation of variables, new cases	
15:00 – 15:25	Victor Enolski, National University “Kyiv-Mohyla Academy” *Modular representation of periods of hyperelliptic integrals*
15:35 – 16:00	Taras Skrypnyk, Universita degli Studi di Milano-Bicocca *Separation of variables in anisotropic models and non-skew-symmetric elliptic r-matrix*
16:00 – 16:25	Thomas Mettler, Goethe Universität Frankfurt GL(2)*-structures in dimension four, H-flatness and integrability*
16:25 – 17:00	Coffee break
17:00 – 17:25	Mats Vermeeren, Technische Universität Berlin *Continuum limits of pluri-Lagrangian systems*
17:35 – 18:00	Tamás F. Görbe, University of Szeged *Compactified trigonometric Ruijsenaars-Schneider systems*
Different aspects	
15:00 – 15:25	Mikhail Tuzhilin, Lomonosov Moscow State University *Splitting of saddle-saddle singularities of integrable Hamiltonian systems*
15:35 – 16:00	Christophe Wacheux, University of Manchester *Convexity of singular affine structures of almost-toric systems*
16:00 – 16:25	Stanislav Nikolaienko, Lomonosov Moscow State University *Topological classification of non-compact 1D Liouville foliations*
16:25 – 17:00	Coffee break
17:00 – 17:25	Thomas Waters, University of Portsmouth *Integrable geodesic flows on tubes and the conjugate locus*
17:35 – 18:00	Sanjay Ramassamy, ENS Lyon *Miquel dynamics for circle patterns*

Friday, 7th of July 2017	
09:30 – 10:20	Jonathan Weitsman, Northeastern University *On the geometric quantization of (some) Poisson manifolds*
10:30 – 11:20	Anton Alekseev, University of Geneva *Poisson brackets dominated by potentials and Gelfand-Zeitlin integrable systems*
11:20 – 11:50	Coffee break
11:50 – 12:20	Konstantinos Efstathiou, University of Groningen *Fractional Hamiltonian monodromy and circle actions*
12:20 –12:55	Stefanella Boatto, Universidade Federal de Rio Janeiro *N-body and N-vortex dynamics on surfaces: curvature and topological contributions*
12:55 – 13:45	Patrick Foulon, CIRM *Integrability and orbital classification of geodesic flows of finsler metrics of constant positive flag curvature*
13:45 – 14:00	Closing
14:00 – 15:00	Lunch

2. Abstracts of the Speakers

Poisson brackets dominated by potentials and Gelfand-Zeitlin integrable systems

Anton Alekseev, University of Geneva
E-mail address: Anton.Alekseev@unige.ch.

Gelfand-Zeitlin integrable systems on coadjoint orbits were discovered by Guillemin and Sternberg. This discovery was inspired by the construction of bases in irreducible representations of the groups $G = SU(n), SO(n)$. It was shown by Berenstein and Kazhdan that Gelfand-Zeiltin cones (and more generally string cones) admit a beautiful and economic description using the potential function Φ on double Bruhat cells.

In the talk, we'll explain that the canonical Poisson bracket π^* on the dual Poisson-Lie group G^* provides a link between the two approaches. In more detail, π^* is dominated by Φ, and the link between π^* and the KKS bracket on the coadjoint orbits is established using the Ginzburg-Weinstein isomorphism.

The talk is based on joint works (some of them in progress) with A. Berenstein, I. Davydenkova, B. Hoffman, and Y. Li.

Algebraic non-integrability of billiards

Michael Bialy, Tel Aviv University
E-mail address: bialy@post.tau.ac.il.

Various approaches have been developed in a very old question by Birkhoff on integrable billiards. Algebraic approach to this question was initiated by S.Bolotin and S.Tabachnikov for Birkhoff?s and Outer billiards respectively. In my talk I am going to discuss recent progress in the question of algebraic integrability. In particular, I shall discuss non-integrability of magnetic billiards.

This is a joint works with Andrey Mironov (Novosibirsk).

Bochner-flat (pseudo-)Kähler metrics, Manakov-type operators and new symmetric spaces

Alexey Bolsinov, Loughborough University
E-mail address: A.Bolsinov@lboro.ac.uk.

In my talk I will be focused on an interesting relationship between integrable systems on semisimple Lie algebras defined by Manakov-type operators and curvature tensors of Kähler metrics of a certain class. This relationship allowed us to complete the local classification of Bochner-flat Kähler metrics of arbitrary signature (for positive definite Kähler metrics, this classification is due to R. Bryant) and to "discover" a new class of symmetric spaces in pseudo-Riemannian geometry.

Equivariant quantization of the Hitchin moduli space
Sergei Gukov, California Institute of Technology
E-mail address: gukov@theory.caltech.edu.

This talk will be a fairly broad review of exploring geometry and topology of the moduli space of Higgs bundles through the equivariant circle action (which acts by a phase on the Higgs field). This approach leads to new invariants of the moduli space of Higgs bundles, the so-called equivariant Verlinde formula, the real and wild versions of the Hitchin character, and the equivariant elliptic genus. The real reason, though, for studying these new invariants is not so much that they contain wealth of useful information about Higgs bundles (they actually do!) but that they have surprising new connections to other problems in math and mathematical physics.

Birkhoff Conjecture for convex planar billiards
Vadim Kaloshin, University of Maryland
E-mail address: vadim.kaloshin@gmail.com.

G.D. Birkhoff introduced a mathematical billiard inside of a convex domain as the motion of a massless particle with elastic reflection at the boundary. A theorem of Poncelet says that the billiard inside an ellipse is integrable, in the sense that the neighborhood of the boundary is foliated by smooth closed curves and each billiard orbit near the boundary is tangent to one and only one such curve (in this particular case, a confocal ellipse). A famous conjecture by Birkhoff claims that ellipses are the only domains with this property. We show a local version of this conjecture - namely, that a small perturbation of an ellipse has this property only if it is itself an ellipse.

This is based on several papers with Avila, De Simoi, G. Huang, Sorrentino.

Celestial mechanics: Aspects of integrability
Andreas Knauf, Friedrich-Alexander-Universität Erlangen
E-mail address: knauf@math.fau.de.

The notion of integrability was introduced in the context of celestial mechanics. We will give an overview of topological and geometric aspects of (non-)integrability, for the n–center and the n–body problem. In particular, it will be shown

- how regularization of the Kepler problem leads to a symplectic manifold that is not a cotangent bundle,
- how non-abelian fundamental groups occur for $n \geq 3$,
- that they give rise to compositions of symplectic Dehn twists,
- that these can be compatible with Gevrey integrability, and
- that stacks of symplectic manifolds may be an adequate model for n-body dynamics.

Desingularizations of Poisson structures of compact type and action-angle coordinates

David Martínez Torres, Pontificia Universidade Catolica do Rio de Janeiro
E-mail address: dfmtorres@mat.puc-rio.br.

A symplectic manifold fibered by isotropic tori induces on its base a regular Poisson structure o a very particular type. Under mild assumptions, this is an example of a so-called (regular) Poisson manifold of compact type.

We will discuss how Poisson structures of compact types can be desingularised into regular ones (at the mild expense of passing to Dirac geometry). This is related to the problem of turning generalised action-angle coordinates into action-angle coordinates.

Of relationships between the Toda chain, QRT maps, Poncelet's porism and the Painlevé VI equation

Emma Previato, Boston University
E-mail address: ep@bu.edu.

The talk will present solutions of the finite Toda chain that arise from Poncelet dynamics. Poncelet's porism theorem can be viewed as a discrete integrable system, arising from the addition of a given point on an elliptic curve, or a QRT map. N.J. Hitchin identified a relationship of Poncelet's porism with algebraic solutions of the Painlevé VI equation. It is known that similarity reduction of integrable hierarchies of PDEs, including the discrete-time case such as the Toda chain, give solutions to Painlevé-type equations. Extensions of these relationships to hyperelliptic curves will be presented, and questions about similarity reductions and Darboux transformations will be posed.

Dynamics of constrained many body problems on sphere and rigid bodies with movable l points

Maria Przybylska, University of Zielona Gora
E-mail address: M.Przybylska@if.uz.zgora.pl.

Recently a lot of work has been devoted to study N materials points in spaces of constant positive curvature S^2 and S^3 interacting according to analogues of Newtonian law of gravitation or Hooke's law of elasticity. Whereas less is known about dynamics of material points on sphere whose motion is subjected to certain holonomic constraints e.g. fixed arc lengths between certain number of material points.

We will show that dynamics of a few material points on sphere S^2 with a certain number of such constraints is equivalent to dynamics of a rigid body with a fixed point with some additional internal degrees of freedom. In particular dynamics of two material points on sphere with fixed arc length between them is equivalent to the dynamics of a rigid body with one fixed point and specific moments of inertia. A system of three points on a sphere constrained such that the distances between two their pairs are fixed is equivalent to a rigid body with one additional point which can move freely along a circle fixed in the body. This last system can be generalized to a rigid body with a fixed point containing a several points which can move along prescribed curves inside this rigid body. Equations of motion of such system are presented and their integrability is analysed. Obtained equations after reduction by their symmetry are Hamiltonian with a certain degenerated Poisson structure. Some integrable cases are identified and non-integrability proofs by means of differential Galois theory are given.

This is a joint work with Andrzej J. Maciejewski.

Pluri-Lagrangian systems, Sklyanin's spectrality property, and billiards in confocal quadrics

Yuri B. Suris, Technische Universität Berlin
E-mail address: suris@math.tu-berlin.de.

We discuss the emerging theory of pluri-Lagrangian systems, as a theory of integrability for systems coming from a variational principle. In particular, in this framework we clarify the origin of the Sklyanin's spectrality property of Bäcklund transformations. This theory is illustrated with the example of commuting billiard maps in confocal quadrics.

Integrability and chaos: coexistence in one system

Dmitry Treschev, Moscow State University
E-mail address: treschev@mi.ras.ru.

I plan to discuss several examples of hamiltonian systems where both integrable and chaotic dynamics occupy open domains in the phase space.

Action-angle coordinates for integrable systems

Pol Vanhaecke, Université de Poitiers
E-mail address: Pol.Vanhaecke@math.univ-poitiers.fr.

I will discuss the existence of local action-angle variables and the obstructions to their global existence in the general context of non-commutative integrable systems on Poisson manifolds.

On the geometric quantization of (some) Poisson manifolds

Jonathan Weitsman, Northeastern University
E-mail address: j.weitsman@neu.edu.

We review geometric quantization in the symplectic case, and show how the program of formal geometric quantization can be extended to certain classes of Poisson manifolds equipped with appropriate Hamiltonian group actions. These include b-symplectic manifolds, where the quantization turns out to be finite dimensional, as well as more singular examples (b^k-symplectic manifolds) where the quantization is finite dimensional for odd k and infinite dimensional, with a very simple asymptotic behavior, where k is even. If time permits we will also discuss some preliminary results for pseudoconvex domains.

This is a joint work with Victor Guillemin and Eva Miranda.

3. Abstracts of the Contributed Talks

Integrable Hénon–Heiles systems on curved spaces
Angel Ballesteros, University of Burgos
E-mail address: angelb@ubu.es.

It is well known that the following multiparametric generalization of the two-dimensional Hénon–Heiles Hamiltonian system

$$\mathcal{H} = \frac{1}{2}(p_1^2 + p_2^2) + \Omega_1\, q_1^2 + \Omega_2\, q_2^2 + \alpha\left(q_1^2 q_2 + \beta\, q_2^3\right),$$

admits only three Liouville integrable cases, namely:
- The Sawada–Kotera Hamiltonian ($\beta = 1/3$, $\Omega_1 = \Omega_2$).
- The Korteweg–de Vries (KdV) case ($\beta = 2$, Ω_1 and Ω_2 arbitrary).
- The Kaup–Kupershmidt case ($\beta = 16/3, \Omega_2 = 16\,\Omega_1$).

In this contribution we present a constant curvature analogue $\mathcal{H}_\kappa$ of the integrable KdV Hénon–Heiles Hamiltonian with $\Omega_2 = 4\Omega_1$ [3]. Our approach is based on the use of the constant Gaussian curvature κ of the underlying spaces as an explicit deformation parameter [2, 3]. This allows us to present this curved Hénon–Heiles Hamiltonian in a unified geometric setting that contains, simultaneously, the spherical and hyperbolic cases, and from which the Euclidean system is obtained in the flat limit $\kappa \to 0$.

In particular, the starting point for the construction will be the anisotropic oscillator on the sphere and the hyperbolic plane that was proposed in [3]. We remark that, under the specific tuning in the frequencies $\Omega_2 = 4\Omega_1$, this system coincides with the well known superintegrable curved 1 : 2 oscillator system obtained in [2]. From the latter, the curved version of the Ramani–Dorizzi–Grammaticos homogeneous potentials [4] can be obtained, and the Hénon-Heiles system arises as the corresponding 'cubic' system within this family [3]. The separability of the curved Hamiltonian in a suitable set of coordinates is demonstrated, and its generalization to the case with arbitrary Ω_1 and Ω_2 is analysed.

Finally, this strategy is also shown to be applicable to the Sawada-Kotera case [2], and the construction of the associated curved Hamiltonian is sketched.

This is a joint work with Alfonso Blasco, Francisco J. Herranz and Fabio Musso.

References

[1] A. Ballesteros, A. Blasco, F.J. Herranz, and F. Musso. *An integrable Hénon-Heiles system on the sphere and the hyperbolic plane.* Nonlinearity, 28 (2015), 3789.

[2] M.F. Rañada and M. Santander. *Superintegrable systems on the 2D sphere S^2 and the hyperbolic plane H^2.* Journal of Mathematical Physics 40 (1999), 5026.

[3] A. Ballesteros, A. Blasco, F.J. Herranz, and F. Musso. *A new integrable anisotropic oscillator on the two-dimensional sphere and the hyperbolic plane.* Journal of Physics A: Mathematical and Theoretical, 47 (2014), 345204.

[4] A. Ballesteros, F.J. Herranz, and F. Musso. *The anisotropic oscillator on the 2D sphere and the hyperbolic plane.* Nonlinearity 26 (2013), 971.
[5] A. Ramani, B. Dorizzi, and B. Grammaticos. *Painlevé conjecture revisited.* Physical Review Letters 49 (1982), 1539.
[6] A. Ballesteros, A. Blasco, and F.J. Herranz. *Integrable Hénon-Heiles systems on curved spaces: the Sawada-Kotera case.* In preparation (2017).

Topology, singularities and integrability in Hamiltonian systems with two degrees of freedom

Sergey Bolotin, Moscow Steklov Mathematical Institute and University of Wisconsin-Madison
E-mail address: bolotin@mi.ras.ru.

We consider the problem of the existence of polynomial in momenta first integrals of Hamiltonian systems with two degrees of freedom on a fixed energy level (conditional Birkhoff integrals). Suppose that the potential energy V on the configuration space M has several singular points of type $V \sim -|q|^{-\alpha_j}$. It is proved that in the presence of Birkhoff conditional integrals on an energy level $H = h > \sup_M V$, we have $\sum \alpha_j \leq 2\chi(M)$. Under certain conditions positiveness of the topological entropy is also proved. Let $A_k = 2 - 2k^{-1}$, and let n_k be the number of singular points with $A_k \leq \alpha_j < A_{k+1}$. It is proved that if M is a closed surface (or certain conditions at infinity are satisfied) and $\sum_{k\leq 2\leq\infty} n_k A_k > 2\chi(M)$, then the system has a compact chaotic invariant set of on any energy level $H = h > \sup_M V$. As an example, the generalized plane n center problem is considered.

This is a joint work with Valery Kozlov (Moscow Steklov Mathematical Institute).

On the construction of monopoles

Harry Braden, University of Edinburgh
E-mail address: hwb@ed.ac.uk.

Although the study of BPS monopoles is now over 30 years old there are still few analytic results known for the Higgs and gauge fields. For su(2) monopoles without spherical or axial symmetry the only known results are for the Higgs field on a coordinate axis for charge 2. By combining integrable systems and twistor constructions we show how the problem becomes algebraic in an appropriate gauge. In pursuing this programme a number of new results have been found. The approach will be illustrated by presenting the general charge 2 fields.

Dunkl–Cherednik operators and quantum Lax pairs

Oleg Chalykh, University of Leeds
E-mail address: o.chalykh@leeds.ac.uk.

A quantum Lax pair for the Calogero–Moser system was first proposed in [1], cf. [2, 3]. Later it was generalized to arbitrary root systems in [4, 5], based on earlier results in the classical case [6, 7, 8]. It was noted in those works that the quantum Lax matrix had a close resemblance to a Dunkl operator, but a precise relationship was not estiblished. More recently, there have also been works [4, 10, 11], pursuing a link between Dunkl operators and quantum Lax operators, but it remained more of an empirical observation.

We will explain a direct conceptual link between Dunkl operators and quantum Lax pairs. In fact, a Lax pair L, A can be associated to any of the commuting quantum Calogero–Moser Hamiltonians, so we obtain a family of compatible quantum Lax pairs. This approach also works for more general Calogero–Moser systems associated with complex reflection groups. Moreover, it can be used in the elliptic case to construct previously unknown quantum Lax pairs.

Replacing Dunkl operators by their q-analogues, known as Cherednik operators, leads to quantum Lax pairs for the relativistic Calogero–Moser system given by the trigonometric Macdonald–Ruijsenaars operators (Koornwinder–van Diejen operators in the BC_n-case). This gives a uniform construction of both classical and quantum Lax pairs for the systems of Ruijsenaars–Schenider type for all root systems, including the Koornwinder–van Diejen system with five coupling parameters. In the latter case, we combine our approach with an idea from [11] to calculate a Lax pair for the Koornwinder–van Diejen system, which remained elusive until now: the best previous result in that direction was a construction of a classical Lax pair [12] for a two-parametric subfamily.

References

[1] H. Ujino, K. Hikami, and M. Wadati. *Integrability of the quantum Calogero-Moser model.* J. Phys. Soc. Japan 61 (1992), 3425.

[2] B.S. Shastri and B. Sutherland. *Super Lax pairs and infinite symmetries in the* $1/r^2$ *system.* Phys. Rev. Lett. 70 (1993), 4029.

[3] D. Bernard, M. Gaudin, F. D.M. Haldane, and V. Pasquier. *Yang-Baxter equation in long-range interacting systems.* J. Phys. A Math. Gen. 26 (1993), 5219.

[4] A.J. Bordner, N.S. Manton, and R. Sasaki. *Calogero-Moser Models V: Supersymmetry, and quantum Lax pair.* Prog. Theor. Phys. 103 (2000), 463–487.

[5] S.P. Khastgir, A.J. Pocklington, and R. Sasaki. *Quantum Calogero-Moser models: integrability for all root systems.* J. Phys. A Math. Gen. 33 (2000), 9033–9064.

[6] I.M. Krichever. *Elliptic solutions of the Kadomtsev-Petviashvili equation and integrable systems of particles.* Funct. Anal. Appl. 14 (1980), 282–289.

[7] E. D'Hoker and D.H. Phong. *Calogero-Moser Lax pairs with spectral parameter for general Lie algebras.* Nucl. Phys. B 530 (1998), 537–610.

[8] A.J. Bordner, E. Corrigan, and R. Sasaki. *Generalised Calogero-Moser models and universal Lax pair operators.* Prog. Theor. Phys. 102 (1999), 499–529.

[9] A.N. Sergeev and A.P. Veselov. *Dunkl operators at infinity and Calogero–Moser systems.* IMRN 21 (2015), 10959–10986.
[10] M.L. Nazarov and E.K. Sklyanin. *Lax operator for Macdonald symmetric functions.* Lett. Math. Phys. 105 (2015), no. 7, 901–916.
[11] M.L. Nazarov and E.K. Sklyanin. *Cherednik operators and Ruijsenaars-Schneider model at infinity.* Preprint, March 2017. Available at: `arXiv:1703.02794[nlin.SI]`.
[12] B.G. Pusztai and T.F. Gorbe. *Lax representation of the hyperbolic van Diejen dynamics with two coupling parameters.* Preprint, March 2016. Available at: `arXiv:1603.06710[math-ph]`.

Fractional Hamiltonian monodromy and circle actions

Konstantinos Efstathiou, University of Groningen
E-mail address: K.Efstathiou@rug.nl.

Standard Hamiltonian monodromy was introduced by Duistermaat as an obstruction to the existence of global action-angle coordinates in integrable Hamiltonian systems [1]. It refers to the monodromy of torus bundles that typically exist in such systems. Fractional Hamiltonian monodromy, introduced by Nekhoroshev, Sadovskií, and Zhilinskií in [5], generalizes standard monodromy by considering not only torus bundles but also more general fibrations with singular fibers.

In this talk we present results concerning both standard and fractional monodromy that were recently obtained in collaboration with Nikolay Martynchuk [3, 4]. It turns out that, in integrable Hamiltonian systems with a Hamiltonian circle action, both standard and fractional monodromy can be solely determined through a careful study of the fixed points of the circle action and their weights. A basic ingredient of this approach is the definition of generalized parallel transport of homology cycles introduced in [2]. These results will be demonstrated in several examples of integrable Hamiltonian systems.

References

[1] J.J. Duistermaat. *On global action-angle coordinates.* Communications on Pure and Applied Mathematics 33 (1980), no. 6, 687–706.
[2] K. Efstathiou and H.W. Broer. *Uncovering fractional monodromy.* Communications in Mathematical Physics 324 (2013), no. 2, 549–588.
[3] K. Efstathiou and N. Martynchuk. *Monodromy of Hamiltonian systems with complexity 1 torus actions.* Journal of Geometry and Physics, in press (2016).
[4] N. Martynchuk and K. Efstathiou. *Generalized Hamiltonian monodromy and parallel transport on Seifert manifolds.* Submitted.
[5] N.N. Nekhoroshev, D.A. Sadovskií, and B.I. Zhilinskií. *Fractional Hamiltonian monodromy.* Annales Henri Poincaré 7 (2006), 1099–1211.

Modular representation of periods of hyperelliptic integrals
Victor Enolski, National University "Kyiv-Mohyla Academy"
E-mail address: venolski@gmail.com.

We consider hyperelliptic curves of higher genera and basis of holomorphic and meromorphic differentials satisfying generalized Legendre relation. Rosenhain formulae expressing a-periods of genus two holomorphic integrals in terms of theta-constants are generalized to higher genera. A representation of a-periods of second kind integrals in terms of theta-constants is found. These results have both theoretical and practical interest and can be used at algebro-geometric integration of finite-dimensional dynamic systems.

Integrable many-body models in action-angle duality from reductions of Heisenberg doubles
László Fehér, University of Szeged and Wigner RCP
E-mail address: lfeher@physx.u-szeged.hu.

I review recent results on the construction of integrable many-body models by Hamiltonian reduction that generalize the derivation of the Calogero and Sutherland systems due to Kazhdan, Kostant and Sternberg [1]. Symplectic reductions are applied to the Heisenberg double of the standard Poisson $SU(N)$ group equipped with two Abelian Poisson algebras. These Abelian algebras are generated by invariants of the underlying symmetry group, which descend to action and position variables of two Liouville integrable many-body models in action-angle duality. The global structure of the reduced phase spaces and qualitative features of the dynamical systems will be also described. The talk is based on the papers [2, 3, 4, 5, 6] and ongoing joint work with Ian Marshall.

References

[1] D. Kazhdan, B. Kostant, and S. Sternberg. *Hamiltonian group actions and dynamical systems of Calogero type*. Comm. Pure Appl. Math. XXXI (1978), 481–507.
[2] L. Fehér and C. Klimcík. *Poisson-Lie interpretation of trigonometric Ruijsenaars duality.* Commun. Math. Phys. 301 (2011), 55–104.
[3] I. Marshall. *A new model in the Calogero-Ruijsenaars family*. Commun. Math. Phys. 338 (2015), 563–587.
[4] L. Fehér and T.F. Görbe. *On a Poisson-Lie deformation of the $BC(n)$ Sutherland system.* Nucl. Phys. B 901 (2015), 85–114.
[5] L. Fehér and T.F. Görbe. *The full phase space of a model in the Calogero- Ruijsenaars family*. J. Geom. Phys. 115 (2017), 139–149.
[6] L. Fehér and I. Marshall. *The action-angle dual of an integrable Hamiltonian system of Ruijsenaars-Schneider-van Diejen type*. arXiv:1702.06514.

Integrability and orbital classification of geodesic flows of finsler metrics of constant positive flag curvature

Patrick Foulon, CIRM
E-mail address: foulon@cirm-math.fr.

We show that the geodesic flow of any Finsler metric of constant positive flag is Liouville integrable. Using this, we show that in dimension 2 the geodesic flow is conjugate to that of the Katok metrics.

The talk is based on a paper, in preparation, joint with, R. Bryant, S. Ivanov, V. Matveev, and W. Ziller.

Poisson algebras and 3D superintegrable Hamiltonian systems

Allan P. Fordy, University of Leeds
E-mail address: **a.p.fordy@leeds.ac.uk.**

Using a Poisson bracket representation, in 3D, of the Lie algebra $\mathbf{sl}(2)$, we first use highest weight representations to embed this in larger Lie algebras. These are then interpreted as symmetry and conformal symmetry algebras of a Hamiltonian, related to a Casimir function.

We then classify the potentials which can be added, whilst remaining integrable and further consider the specialisation to super-integrable cases. The automorphisms of the symmetry algebra of the kinetic energy are extended to the full Poisson algebra, enabling us to build the full set of Poisson relations.

Compactified trigonometric Ruijsenaars-Schneider systems

Tamás F. Görbe, University of Szeged
E-mail address: tfgorbe@physx.u-szeged.hu.

Recently, new forms of the compactified trigonometric Ruijsenaars-Schneider model have been discovered at the level of classical mechanics by Fehér and Kluck. These models describe N particles moving on a circle whose completed center-of-mass phase space is the complex projective space $\mathbb{CP}^{N-1}$ with the Fubini-Study form. In this talk, we present the quantum version of these integrable systems and solve the corresponding eigenvalue problem by generalising van Diejen and Vinet's earlier results.

This is a joint work with Martin Hallnäs.

On recent progress in semitoric systems. Taylor series invariant for coupled angular momenta and coupled spin oscillators, a new system with 2 focus-focus points, and vertical almost toric systems

Sonja Hohloch, Universiteit Antwerpen, Belgium
E-mail address: sonja.hohloch@uantwerpen.be.

Roughly, a semi-toric system on a compact 4-dimensional manifold consists of two Poisson-commuting Hamiltonian flows one of which is periodic. Thus the flow parameters induce an $S^1 \times \mathbb{R}$-action on the manifold. Under certain assumptions on the singularities, semi-toric systems have been classified by Pelayo & Vu Ngoc by means of five invariants.

Two of these invariants, namely the so-called Taylor series invariant and the twisting index, are some-what 'mysterious' due to lack of intuition and/or numerical evidence. We hope to amend this by

1) Identifying the Taylor series invariant somehow with the Birkhoff normal form and computing it for the coupled angular momenta and the coupled spin oscillators. This is an ongoiong project with J. Alonso Fernandez and H. Dullin.
2) Presenting a new semitoric system with two focus-focus singularities. We suspect that the twisting index is in fact some kind of Dehn twist. This is an ongoing project with J. Palmer.

Moreover, there is the new class of vertical almost toric systems —a generalization of semi-toric systems 'compatible' with taking subsystems, but enjoying still many pleasant properties of semi-toric systems. Vertical almost toric systems are in particular suited for surgeries. This is a joint work with S. Sabatini, D. Sepe, and M. Symington.

If wished by the organisers, I can give an overview talk over the progress/results in those topics or focus my talk on one of them.

On smooth almost direct product decompositions of non-degenerate singularities

Anton Izosimov, University of Toronto and Max Planck Institute for Mathematics
E-mail address: izosimov@math.utoronto.ca.

In 1996 Nguyen Tien Zung proved that any non-degenerate singularity of an integrable Hamiltonian system is homeomorphic to an almost direct product of standard fibrations of four types: regular, elliptic, hyperbolic, and focus-focus. It is quite obvious that such a decomposition is in general *not symplectic*. However, Zung conjectured that this decomposition can be made *smooth*.

The goal of my talk is to explain why the smooth version of Zung's theorem is not true. Namely, I will consrtuct an example of a singularity which is topologically a direct product but is indecomposable in the smooth category. The construction is based on the existence of non-trivial smooth deformations in the focus-focus case.

The talk is based on joint work with A. Bolsinov.

The role of homographic dynamics in atomic and molecular systems
Charles Jaffé, West Virginia University
E-mail address: cjaffe@wvu.edu.

Homographic dynamics are known to play an important role in many Newtonian systems. They are regularly used in the design of the orbit of space craft. Much less is known about the occurrence, existence and role that the homographic dynamics play in Coulombic systems. In this talk I will discuss the role of the homographic dynamics in helium atom He and in the hydrogen cation H_2^+.

Billiards on constant curvature spaces
Božidar Jovanović, University of Belgrade
E-mail address: bozaj@mi.sanu.ac.rs.

It is well known that the billiards within ellipsoids are the only known integrable billiards with smooth boundary in constant curvature spaces [1, 3, 7]. The elliptical billiards in pseudo-Euclidean spaces are also integrable [6, 2]. We will try to present all these integrable models through a unified perspective, within the framework of the virtual billiard dynamic (see [5]), where, in contrast to the usual billiards, the incoming velocity and the velocity after the billiard reflection can be at opposite sides of the tangent plane at the reflection point. In the symmetric case we prove noncommutative integrability of the system and give a geometrical interpretation of integrals, an analog of the classical Chasles and Poncelet theorems and we show that the virtual billiard dynamics provides a natural framework in the study of billiards within quadrics in projective spaces, in particular of billiards within ellipsoids on the sphere $\mathbb{S}^{n-1}$ and the Lobachevsky space $\mathbb{H}^{n-1}$. The skew-hodograph mapping (see [7]) for the billiards on the Lobachevsky space and its relation with Heisenberg system on the light–like cone (see [4]) is also given. The results are obtained jointly with Vladimir Jovanović, University of Banja Luka.

References

[1] С. В. Болотин, *Интегрируемые бильярды на поверхностях постоянной кривизны*, Мат. Заметки 51 (1992), no. 2, 20–28, (Russian); English translation:
S.V. Bolotin. *Integrable billiards on constant curvature surfaces.* Math. Notes 51 (1992), no. 1–2, 117–123.

[2] V. Dragović and M. Radnović. *Ellipsoidal billiards in pseudo-euclidean spaces and relativistic quadrics.* Adv. Math. **231** (2012), 1173–1201, `arXiv:1108.4552`.

[3] Ю. Н. Федоров, *Эллипсоидальный биллиард с квадратичным потенциалом*, Функц. анализ и его прилож. 35 (2001), no. 3, 48–59, 95–96 (Russian); English translation: Yu.N. Fedorov. *An ellipsoidal billiard with quadratical potential.* Funct. Anal. Appl. **35** (2001), no. 3, 199–208.

[4] B. Jovanović. *Heisenberg model in pseudo–Euclidean spaces.* Regular and Chaotic Dynamics 19 (2014), no. 2, 245–250, arXiv:1405.0905 [nlin.SI].

[5] B. Jovanović and V. Jovanović. *Geodesic and billiard flows on quadrics in pseudo–Euclidean spaces: L–A pairs and Chasles theorem.* International Mathematics Research Notices **15** (2015), no. 15, 6618–6638, arXiv:1407.0555.

[6] B. Khesin and S. Tabachnikov. *Pseudo-Riemannian geodesics and billiards.* Adv. Math. **221** (2009), 1364–1396, `arXiv:math/0608620`.

[7] J. Moser and A. Veselov. *Discrete versions of some classical integrable systems and factorization of matrix polynomials.* Comm. Math. Phys. 139 (1991), 217–243.

Bifurcations of the Liouville foliations for the Kovalevskaya case on so(4)

Vladislav Kibkalo, Moscow State University
E-mail address: v.a.kibkalo@gmail.com.

Consider the six-dimensional space $\mathbb{R}^6(\mathbf{J}, \mathbf{x})$ and the following one-parameter family of Poisson brackets depending on the real parameter κ:

$$\{J_i, J_j\} = \varepsilon_{ijk} J_k, \quad \{J_i, x_j\} = \varepsilon_{ijk} x_k, \quad \{x_i, x_j\} = \kappa \varepsilon_{ijk} J_k.$$

These brackets have two Casimir functions:

$$f_1 = x_1^2 + x_2^2 + x_3^2 + \kappa(J_1^2 + J_2^2 + J_3^2), \qquad f_2 = x_1 J_1 + x_2 J_2 + x_3 J_3.$$

I.V. Komarov in his paper [3] showed that the Kovalevskaya integrable case in rigid body dynamics can be included in a one-parameter family of integrable Hamiltonian systems on this pencil of Lie algebras $\mathrm{so}(4) - \mathrm{e}(3) - \mathrm{so}(3,1)$. The Kovalevskaya top was realized as a system on Lie algebra $\mathrm{e}(3)$. The Hamiltonian H and first integral have the following form:

$$H = J_1^2 + J_2^2 + 2J_3^2 + 2c_1 x_1,$$

$$K = (J_1^2 - J_2^2 - 2c_1 x_1 + \kappa c_1^2)^2 + (2J_1 J_2 - 2c_1 x_2)^2,$$

where c_1 is an arbitrary constant.

In the case of $\varkappa > 0$, $a > 0$ the common level surfaces of Casimir functions $M^4_{a,b} = \{f_1 = a, f_2 = b\}$ are compact orbits of coadjoint representation and symplectic leaves of the Lie-Poisson bracket. Every regular $M^4_{a,b}$ has a structure of Liouville foliation. Every two-dimensional Liouville torus is a closure of trajectories of this system. In previous papers [2] the bifurcation diagrams were constructed. Nondegenerate critical points of the rank 0 also were described.

One approach of topological analysis of integrable Hamiltonian systems allows us to describe regular Liouville foliations on 3-dimensional isoenergy surface

$$Q^3_{a,b,h} = \{f_1 = a, f_2 = b, H = h\}$$

using Fomenko-Zieschang invariant [1]. These invariants for all regular Liouville foliations for this system will be presented in our talk. We will also discuss the topology and bifurcations of Liouville foliations for this system.

References

[1] A.T. Fomenko and A.V. Bolsinov. *Integrable Hamiltonian systems: geometry, topology, classification.* CRC Press, 2004.

[2] I.K. Kozlov. *The topology of the Liouville foliation for the Kovalevskaya integrable case on the Lie algebra so(4).* Sbornik: Mathematics 205 (2014), no. 4, 532–572.

[3] I.V. Komarov. *Kowalewski basis for the hydrogen atom.* Teor. Mat. Fiz. 47 (1981), no.1, 6772; English transl. in Theoret. and Math. Phys. 47 (1981), no. 1, 320–324.

Singular points of Nijenhuis operator and left-symmetric algebras

Andrei Konyaev, Moscow State University
E-mail address: maodzund@ya.ru.

The Nijehuis operator is an (1,1)-tensor field with zero Nijenhuis torsion. Such operators appear in different fields of geometry and mechanics.

In semisimple case the structure of such field around the singular point is rather trivial. But it turns out, that the singular points of such operators the situation becomes way more interesting. The author discovered, that tangent space to the singular points of such operators posses the natural structure of left-symmetric algebra (LSA). Such algebras were introduced in different context by E. Vinberg.

It two dimensional case these algebras give a complete solution to the linearization problem for Nijenhuis field. They also provide insight for global topological structure of such fields.

Local non-existence of polynomial integrals for geodesic flows

Boris Kruglikov, University of Tromsø
E-mail address: boris.kruglikov@uit.no.

In the smooth category the geodesic flow of a Riemannian metric is locally integrable. In the analytic category this is generically no longer true. The reason for this fact is overdeterminacy for the system describing polynomial integrals of a fixed degree. While the whole set of solvability conditions is too complicated, I will explain a simple trick that allows to verify that for generic metrics this system has no solutions.

With this approach the regularity of the problem is minimal possible: the space of metrics with no solutions is open dense in the C^2-topology. Time permitting I will also explain how this result applies in tensor tomography.

The talk is based on the joint work with Vladimir Matveev: Nonlinearity 29 (2016), 1755–1768.

Continuous orbital invariants of integrable Hamiltonian systems

Elena Kudryavtseva, Moscow State University
E-mail address: ekudr@list.ru.

We study integrable Hamiltonian systems with 2 degrees of freedom on regular compact isoenergy 3-manifolds. Such a system is given by a pair (B, F) of a closed 2-form B without zeros and a Bott function F (called the first integral) on a compact 3-manifold Q endowed with a volume form, such that $d(FB) = 0$.

A.Bolsinov and A.Fomenko constructed in 1995, under some additional assumptions, a complete set $\{I_k\}$ of orbital invariants $I_k \colon \{(B, F)\} \to \mathbb{R}$ of such integrable systems. From this classification it turned out that any orbital invariant of generic integrable systems is “trivial”, i.e. it can be expressed in terms of local extremes of rotation functions on one-parameter families of invariant tori. Here an integrable system is called *generic* if the values of the first integral F on its critical submanifolds are pairwise distinct. Bolsinov and Fomenko posed the question which of (nontrivial) orbital invariants are “stable” under integrable perturbations, i.e. can be *continuousely extended* to a neighbourhood of the given class of (non-generic) integrable systems (clearly, the trivial invariants are stable). A simplified question can be formulated as follows. Suppose that two (non-generic) integrable systems are given, that have the same topology of the foliation by invariant submanifolds (for simplicity, have the same first integral). Is it possible to make these systems orbitally equivalent via small integrable perturbations, or some orbital invariants exist that can serve as an obstruction to this scenario?

In our talk, we answer the above questions. We show that the key rôle is played by the topology of the singular invariant fiber (i.e. of the integral submanifold different from torus), namely, by the following property. It is known that near a singular invariant fiber the dynamical system always admits a cross-section, which is a 2-manifold with boundary. The key property is the genus of the cross-section. If the genus equals 0 then there are no nontrivial *continuous* orbital invariants $I : \{(B, F)\} \to \mathbb{R}$. However, if the genus is positive, such invariants can appear, and we show which of the orbital invariants have this property in some genus 1 cases.

This work was supported by the Russian Science Foundation grant (project No.17-11-01303).

Normal forms of 2-dimensional metrics with projective vector fields I

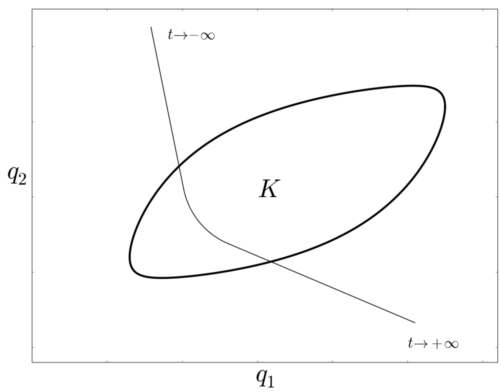
Gianni Manno, Politecnico di Torino
E-mail address: giovanni.manno@polito.it.

In 1882 Sophus Lie posed the problem of classifying 2-dimensional (pseudo-) Riemannian manifolds admitting one or several projective vector fields, i.e., vector fields whose local flow sends geodesics to geodesics (viewed as unparametrized curves). Here I explain how to arrive to normal forms in the case of metrics admitting 2 or more projective vector fields. Then, I will introduce the (linearizable) system of partial differential equations that describes metrics admitting only one projective vector field.

Knauf's topological degree and scattering monodromy

Nikolay Martynchuk, University of Groningen
E-mail address: n.martynchuk@rug.nl.

Consider a Hamiltonian system $\dot{q} = p$ and $\dot{p} = -\partial V$, where the potential function $V(q) \to 0,\ \|q\| \to \infty$. If the decay is sufficiently fast, then the dynamics becomes 'free' at infinity, making it possible to compare a given distribution of particles at $t = -\infty$ (initial data) with their final distribution at $t = +\infty$ (asymptotic data).



For a non-trapping energy $E > 0$ this results in the so-called scattering map and its topological degree $\deg(E)$ [4].

In this talk we will discuss $\deg(E)$ in connection with another topological invariant of scattering, called scattering monodromy [1–3]. We will demonstrate that — for rotationally symmetric potentials — scattering monodromy is given by the jump of the degree $\deg(E)$, which appears when the non-trapping energy E goes from low to high values.

This is a report on my joint work with Holger Waalkens [5].

References

[1] L. Bates and R. Cushman. *Scattering monodromy and the A1 singularity.* Central European Journal of Mathematics 5 (2007), no. 3, 429–451.

[2] H. Dullin and H. Waalkens. *Nonuniqueness of the phase shift in central scattering due to monodromy.* Phys. Rev. Lett. 101 (2008).

[3] K. Efstathiou, A. Giacobbe, P. Mardešić, and D. Sugny. *Rotation forms and local Hamiltonian monodromy.* Submitted (2016).

[4] A. Knauf. *Qualitative aspects of classical potential scattering.* Regul. Chaotic Dyn. 4 (1999), no. 1, 3–22.

[5] Nikolay Martynchuk and Holger Waalkens. *Knauf's degree and monodromy in planar potential scattering.* Regular and Chaotic Dynamics 21 (2016), no. 6, 697–706.

GL(2)*-structures in dimension four, H-flatness and integrability*

Thomas Mettler, Goethe Universität Frankfurt
E-mail address: mettler@math.uni-frankfurt.de.

We show that torsion-free four-dimensional GL(2)-structures are flat up to a coframe transformation with a mapping taking values in a certain four-dimensional subgroup of the group of upper triangular matrices in $SL(4,\mathbb{R})$. In addition, we show that the relevent PDE system is integrable in the sense that it admits a dispersionless Lax-pair.

This is a joint work with W. Kryński.

References

[1] T. Mettler and K. Wojciech. GL(2)*-structures in dimension four, H-flatness and integrability.* (2016), arXiv:1611.08228.

Topological classification of non-compact 1D Liouville foliations

Stanislav Nikolaienko, Lomonosov Moscow State University
E-mail address: nikostas@mail.ru.

We consider some new aspects of the problem of the topological classification of completely integrable Hamiltonian systems. This problem has been extensively studied by A. Fomenko, S. Matveev, Nguyen Tien Zung, J. Duistermaat, V. Matveev, L. Lerman and others in the case when all the leaves of the corresponding Liouville foliations are compact. However, very little is known in the non-compact case. In our work we study bifurcations of "non-compact" Liouville foliations for the simplest case of Hamiltonian systems with one degree of freedom. The leaves of such foliations are level sets of smooth functions (Hamiltonian functions) on smooth non-compact 2-dimensional manifolds. Under some natural conditions the foliations of this type can be classified up to topological equivalence in terms similar to those in the compact case. In particular, we suggest two algorithms for enumerating all the bifurcations appearing in such foliations.

One of these algorithms involves a generalization of the notion of an f-graph suggested by A.A. Oshemkov in [1].

References

[1] A.A. Oshemkov. *Morse functions on two-dimensional surfaces. Encoding of singularities.* Proc. Steklov Inst. Math. 205 (1995), 119–127.

Bifurcation diagram for Adler – van Moerbeke integrable case

Andrey Oshemkov, Lomonosov Moscow State University
E-mail address: a@oshemkov.ru.

The Adler – van Moerbeke integrable case is an integrable system defined by the Euler equations on the Lie algebra $so(4)$. The Hamiltonian of this system is quadratic, and the additional integral is a polynomial of degree four. This integrable case is connected with various interesting algebraic constructions. For example, it can be obtained with the help of the "argument shift method" applied to the Lie algebra $so(4)$ considered as the normal form of the exceptional simple Lie algebra g_2. This method for constructing complete involutive families of polynomials on Lie algebras was developed by A. S. Mischenko and A. T. Fomenko in 1978-79. Later, in 1984, M. Adler and P. van Moerbeke found this integrable case investigating left-invariant metrics on $SO(4)$ that are algebraically completely integrable. Also, in 1986, A. G. Reyman and M. A. Semenov-Tian-Shansky found a Lax pair for this integrable system.

The topological analisys of the Adler – van Moerbeke integrable case was recently done by A. A. Oshemkov, S. V. Sokolov, P. E. Ryabov. In particular, we describe its singularities, their types, and the bifurcation diagram of the corresponding momentum mapping. Note that there are several known integrable cases on $so(4)$ with quartic additional integrals, and the obtained results show qualitative differences of the Adler – van Moerbeke integrable case from them.

New constructions of semitoric integrable systems

Joseph Palmer, Rutgers, the State University of New Jersey
E-mail address: jpalmermath@gmail.com.

Semitoric integrable systems, which generalize toric integrable systems in dimension four, were recently classified in terms of five invariants by the work of Pelayo-Vũ Ngọc, but until now there have been relatively few concrete examples of such systems. We present several new examples of semitoric systems (including one with two focus-focus singularities) and outline a general construction for producing new semitoric integrable systems.

This is a joint work with Y. Le Floch and S. Hohloch.

On local bisymplectic realizations of compatible Poisson brackets

Andriy Panasyuk, University of Warmia and Mazury
E-mail address: panas@matman.uwm.edu.pl.

In a seminal paper "The local structure of Poisson manifold" (1983) A. Weinstein proved that for any Poisson manifold (M, P) there exists a local symplectic realization, i.e., nondegenerate Poisson manifold (M', P') and a local surjective submersion $f\colon M' \to M$ with $f_*P' = P$. Global aspects of this problem were afterwards intensively studied as they are related to the theory of symplectic and Poisson grupoids, to the integration problem of Lie algebroids, and to different quantization schemes.

In this talk I will discuss a problem of local simultaneous realization of two compatible Poisson structures by means of two nondegenerate ones. Note the following essential difference between the two realization problems: there is only one local model of the nondegenerate Poisson bivector P' given by the Darboux theorem and there are many local models of bisymplectic bihamiltonian structures. Thus, besides the problem of existence it is important to understand how many nonequivalent realizations there are in the second case.

Miquel dynamics for circle patterns

Sanjay Ramassamy, ENS Lyon
E-mail address: ramassam@math.brown.edu.

A circle pattern with the combinatorics of the square grid is an embedding of the square grid such that every face admits a circumcircle. Using Miquel's six circles theorem, we define a discrete dynamical system on the space of such circle patterns on a flat torus. I will describe some properties of this dynamics that suggest it may be an integrable system.

Integrability properties of the geodesic equation in sub-Riemannian spaces

Witold Respondek, Normandie Université
E-mail address: witold.respondek@insa-rouen.fr.

Keywords: Sub-Riemannian geometry, Liouville integrability, B-integrability, differential Galois theory, Pontryagin Maximum Principle, adjoint equation, control of quantum Systems, optimal control

The aim of this contribution is to study integrability of the geodesic equation (adjoint equation) in sub-Riemannian problems and to show usefulness of the Morales-Ramis theorem in proving nonintegrability. A *sub-Riemannian manifold*, shortly SR-manifold, is a triple $(M, \mathcal{D}, B)$, where M is a smooth manifold, $\mathcal{D}$ is a smooth distribution of rank m on M, and B a smoothly varying positive definite bilinear form on $\mathcal{D}$, that is, a smoothly varying scalar product on $\mathcal{D}$.

Sub-Riemannian geodesics are curves that are tangent to the distribution $\mathcal{D}$ and minimize the SR-distance. Using the Pontryagin Maximum Principle PMP the geodesic equation is represented as a Hamiltonian system on the cotangent bundle T^*M of the underlying manifold M. In the case of Lie groups, fibers-components of the geodesic equation, called the adjoint equation, can be separated from those on the base. We study both Liuoville- and B-integrability properties of the geodesic equation in three cases.

First, we consider the adjoint equation for SR-structures on 3-dimensional homogeneous spaces. We prove that B-integrability of the adjoint equation holds if and only if the underlying homogeneous SR-manifold is symmetric or, equivalently, the optimal controls of the corresponding optimal problem are elliptic functions of time.

Second, we study the nilpotent approximations of generic 3-dimensional SR-structures. It is known that in the contact and in the Martinet case, the geodesic equation is Liuoville integrable. We prove that the next degenaration, called the tanget case, is not Liuoville integrable.

Third, we consider the adjoint equation for a class of SR-structures on SO(n) that describes optimal controls for the minimal energy laser-induced population transfer for n level quantum systems. We prove that the adjoint equqtion is not Liuoville integrable for $n = 4$ and not B-integrable for $n \geq 4$. The first case is studied by a direct analysis, in proving nonintegrability for the second and third case we use the differential Galois theory and, in particular, Morales-Ramis theorem.

This is a joint work with Andrzej Maciejewski.

Matrix Painlevé hierarchies

Pilar Ruiz Gordoa, Universidad Rey Juan Carlos
E-mail address: pilar.gordoa@urjc.es.

We consider the construction of hierarchies of matrix ordinary differential equations, analogous to scalar Painlevé hierarchies. We give as examples a matrix first Painlevé hierarchy and a matrix second Painlevé hierarchy. We also discuss the derivation of properties such as Bäcklund and auto-Bäcklund transformations.

Separation of variables in anisotropic models and non-skew-symmetric elliptic r-matrix

Taras Skrypnyk, Universita degli Studi di Milano-Bicocca
E-mail address: taras.skrypnyk@unimib.it.

Using the classical r-matrix approach [1] we resolve a problem of separation of variables for all the classical integrable hamiltonian systems possessing Lax matrices that satisfy linear Poisson brackets with non-skew-symmetric, non-dynamical elliptic $so(3) \otimes so(3)$-valued classical r-matrix. Using the corresponding Lax matrices we present a general form of the "separating functions" $B(u)$ and $A(u)$ that generate the coordinates and the momenta of separation for the associated models [2]. We consider several examples and perform the separation of variables for the classical anisotropic Euler's top, Steklov-Lyapunov model of the motion of anisotropic rigid body in the liquid [3]-[4] (see also [5]), two-spin generalized Gaudin model [6] and "spin" generalization of Steklov-Lyapunov model [2].

References

[1] E. Sklyanin. *Progress of Theoretical Physics*. Supplement 35(118) (1995).
[2] T. Skrypnyk. *Lett Math Phys* (2016). doi:10.1007/s11005-016-0920-0.
[3] V. Steklov. *On the motion of a rigid body in a fluid*. Kharkov (1903).
[4] A. Lyapunov. Soob. Khark. Matem. Ob. 2, 4 (1893), no. 1–2, 81–88
[5] Y. Fedorov and I. Basak. Regul. Chaotic Dyn. 16 (2011), no. 3–4, 374–395.
[6] T. Skrypnyk. J. Geom. Phys. 57 (2006), no. 1, 53–67.

Geometric quantisation of semitoric integrable systems and almost toric manifolds

Romero Solha, PUC-Rio

E-mail address: romerosolha@gmail.com.

This talk address the focus-focus contribution to real geometric quantisation (where real polarisations are given by integrable systems), with the quantisation of K3 surfaces serving as model examples and motivation. Near a singular Bohr-Sommerfeld focus-focus fibre of an integrable system in dimension four, the zeroth and first cohomology groups computing geometric quantisation are trivial, but the second cohomology group is not; the latter is actually infinite dimensional. As a consequence, the real geometric quantisation of semitoric integrable systems and almost toric manifolds are computed.

This is a joint work with Eva Miranda and Francisco Presas

Integrability of geodesic flow on nilpotent Lie groups of pseudo-H-type

Daisuke Tarama, Ritsumeikan University

E-mail address: dtarama@fc.ritsumei.ac.jp.

This talk deals with the complete integrability of geodesic flow on step-two nilpotent Lie groups of pseudo-H-type with respect to a left-invariant pseudo-Riemannian metric.

In 1972, Vergne has shown the existence of a maximal set of Poisson commuting polynomials on nilpotent Lie algebras. However, there seems to be no explicit construction of the constants of motion for geodesic flow on nilpotent Lie groups except for Heisenberg groups (Kocsard, Ovando, and Reggiani, 2016).

In this talk, some sets of Poisson commuting first integrals are explicitly constructed to show the complete integrability of the geodesic flow with respect to a left-invariant metric on step-two nilpotent Lie groups of pseudo-H-type, which have been introduced by Ciatti in 2000. The integrability on the corresponding nilmanifolds is also considered.

This is a joint work with Wolfram Bauer (Leibniz Universität Hannover).

On integrability of the Killing equation
Kentaro Tomoda, Kobe University
E-mail address: k-tomoda@stu.kobe-u.ac.jp.

Killing tensor fields have been thought of as describing hidden symmetry of space(-time) since they are in one-to-one correspondence with polynomial first integrals of geodesic equations. Many problems in classical mechanics can be formulated as geodesic problems in curved spaces and spacetimes, and thus solving the defining equation for Killing tensor fields (the Killing equation) is a powerful way to integrate the equations of motion. In this talk we attempt to formulate the integrability conditions of the Killing equation, which serve to put an upper bound on the number of linearly independent solutions and also to restrict the possible forms of solutions tightly. To this end, we first show the prologation for the Killing equation in a manner that uses Young symmetrizers. Then, using the prolonged equations, we provide the integrability conditions explicitly.

This is a joint work with Tsuyoshi Houri and Yukinori Yasui.

Splitting of saddle-saddle singularities of integrable Hamiltonian systems
Mikhail Tuzhilin, Lomonosov Moscow State University
E-mail address: mtu93@mail.ru.

Consider an integrable Hamiltonian system with two degrees of freedom. Two integrals on 4-dimensional manifold determine a momentum map. Consider the momentum map foliation on a neighbourhood, so-called 4-dimensional singularity, of a zero rank singular value. Fomenko hypothesis says that if all singular points of the momentum map are non-degenerate then the foliation of the singularity is completely defined by the foliation of the singularity boundary up to Liouville equivalence. It turns out that if the singular value has focus-focus, or center-center, or center-saddle type then the hypothesis is true, but if the type is saddle-saddle then it is not, see [4].

The main problem we set is to find the connections between the boundary foliation and the singularity foliation in the saddle-saddle case. It turns out that the boundary foliations of low complexity singularities define the possibility of splitting corresponding singularities. In this report we will give definitions of splitting in our case and describe this result.

This report requires some knowledge of the atoms and molecules theory, see [1], and the Zung theorem [2]. See [3] and [4] for previous results in this field.

References

[1] A.V. Bolsinov and A.T. Fomenko. *Integrable Hamiltonian systems.* Geometry, Topology and Classification// CRC Press. 2004.

[2] T.Z. Nguen. *Symplectic topology of integrable Hamiltonian systems, I: Arnold–Liouville with singularities.* Compositio Mathematica 101 (1996), 179–215.

[3] M.A. Tuzhilin. *Invariants of four- and three-dimensional singularities of integrable systems.* Doklady Mathematics 93(2), 186–189.

[4] M.A. Tuzhilin. *Singularities of integrable Hamiltonian systems with the same boundary foliation. An infinite series.*. Moscow University Mathematics Bulletin 71(5) (2016), 185–190.

Continuum limits of pluri-Lagrangian systems

Mats Vermeeren, Technische Universität Berlin
E-mail address: vermeeren@math.tu-berlin.de.

The *Lagrangian multiform* or *pluri-Lagrangian* structure of integrable quad equations is well-studied by now, see for example [1, 2, 3, 7]. It was observed early in these studies that the lattice parameters of a discrete pluri-Lagrangian system may play the role of independent variables in a corresponding continuous pluri-Lagrangian system of non-autonomous differential equations.

This contribution presents a different connection between discrete and continuous pluri-Lagrangian systems, where the continuous variables interpolate the discrete ones. In this context, the lattice parameters describe the size and shape of the mesh on which the discrete system lives, and thus they disappear in the continuum limit. The continuous systems found this way are hierarchies of autonomous differential equations. Pluri-Lagrangian systems of this type were studied independently of the discrete case in [4].

Some continuum limits in this sense can be found in the literature, most notably in [6], where the lattice potential KdV equation is shown to produce the potential KdV hierarchy in a suitable limit. The complicated double limit procedure from that work can be presented in a simplified form using Miwa variables. In this form, the procedure is easily adapted to some other quad equations, at least on the level of the equations themselves.

On the level of the pluri-Lagrangian structure, the problem is essentially that of Lagrangian interpolation of discrete variational systems. This was studied in [5] because of its relevance in numerical analysis for backward error analysis of variational integrators. We build on the ideas from that work to construct a pluri-Lagrangian structure for several hierarchies of differential equations that appear as continuum limits of quad equations.

References

[1] A.I. Bobenko and Yu.B. Suris. *On the Lagrangian structure of integrable quad-equations.* Lett. Math. Phys. 92 (2010), no. 1, 17–31.

[2] R. Boll, M. Petrera, and Yu.B. Suris. *What is integrability of discrete variational systems?* Proc. Royal Soc. A, 470 (2014), no. 2162, 20130550.

[3] S. Lobb and F. Nijhoff. *Lagrangian multiforms and multidimensional consistency.* Journal of Physics A: Mathematical and Theoretical 42 (2009), no. 45, 454013.

[4] Yu.B. Suris and M. Vermeeren. *On the variational structure of integrable hierarchies.* Advances in Discrete Differential Geometry, Ed. A.I. Bobenko, Springer (2016): 347–378.

[5] M. Vermeeren. *Modified equations for variational integrators.* (2015), `arXiv:1505.05411`.

[6] G.L. Wiersma and H.W. Capel. *Lattice equations, hierarchies and Hamiltonian structures.* Physica A: Statistical Mechanics and its Applications 142 (1987), no. 1, 199–244.

[7] P. Xenitidis, F. Nijhoff, and S. Lobb. *On the Lagrangian formulation of multidimensionally consistent systems.* Proc. R. Soc. A 467 (2011), no. 2135, 3295–3317.

Normal forms of 2-dimensional metrics with projective vector fields II

Andreas Vollmer, INdAM - Politecnico di Torino

E-mail address: andreas.d.vollmer@gmail.com.

In 1882, Sophus Lie posed the problem of classifying 2-dimensional (pseudo-) Riemannian manifolds with projective vector fields.

Taking up the talk of Gianni Manno, my talk will introduce the projective classes that cover metrics with one essential (i.e. non-homothetic) projective vector field. Then, after clarifying the properties of metrics with a homothetic vector field, the normal forms of metrics with exactly one essential projective vector field can be found.

Convexity of singular affine structures of almost-toric systems

Christophe Wacheux, University of Manchester

E-mail address: christophe.wacheux.math@gmail.com.

On a symplectic manifold (M^{2n}, ω), toric systems are integrable Hamiltonian systems whose time-1 flow yields a $\mathbb{T}^n$-action. The Atiyah - Guillemin & Sternberg theorem [Ati82], [GS82], [GS84] uses on the affine structure of the base space of the associated Lagrangian fibration to prove that the image the moment map is a convex polytope in $\mathbb{R}^n$.

In *almost-toric* systems, one allows so-called focus-focus singularities to occur, which in turn destroys some of the S^1-actions. For these almost-toric systems, the affine structure becomes singular and the image of the moment map is not a convex polytope of $\mathbb{R}^n$ in general. We will discuss the notion of *intrinsic convexity* of the base space with respect to its (singular) affine structure and give local and global convexity results for the base space.

This is a joint project with Tudor Ratiu and Nguyen Tien Zung.

References

[Ati82] M.F. Atiyah. *Convexity and commuting hamiltonians*. Bulletin of the London Mathematical Society 14 (1982), no. 1, 1–15.

[GS82] V. Guillemin and S. Sternberg. *Convexity properties of the moment mapping.* Inventiones Mathematicae 67 (1982), 491–513, doi:10.1007/BF01398933.

[GS84] V. Guillemin and S. Sternberg. *Convexity properties of the moment mapping. ii.* Inventiones Mathematicae 77 (1984), 533–546, doi:10.1007/BF01388837.

Integrable geodesic flows on tubes and the conjugate locus

Thomas Waters, University of Portsmouth

E-mail address: thomas.waters@port.ac.uk.

We construct new examples of surfaces with integrable geodesic flow by considering tubular sub-manifolds generated by curves. We also study the focal sets of tubes about points in surfaces, also known as the conjugate locus, and their bifurcations. We prove a simple relationship between a topological and a geometrical property of the conjugate locus (viz. the rotation index and the number of cusps) in the case of convex surfaces.

4. Abstracts of the Posters

Integrable structure in Liouville gravity correlation numbers

Konstantin Aleshkin, SISSA, Landau ITP
E-mail address: kaleshkin@sissa.it.

One of the approaches to the two-dimensional quantum gravity uses matrix models formalism and gives correlation numbers in terms of a specific tau function of some integrable hierarchy. Another approach uses direct evaluation of the path integral in conformal gauge and is called Liouville gravity. It was noticed that these two approaches give very similar results, though no general statements exist. Comparison is difficult because explicit computations on the side of the Liouville gravity involve complicated integrals over a moduli space of curves starting from the four-point correlation numbers on a sphere. In the talk I will discuss the computation of the four-point correlation numbers and comparison between the two approaches.

Taylor series symplectic invariant of semi-toric coupled angular momenta

Jaume Alonso Fernandez, University of Antwerp
E-mail address: jaume.alonsofernandez@uantwerpen.be.

We explicitly compute the higher order terms of the Taylor series symplectic invariant of an integrable semi-toric system given by the coupling of two angular momenta in a non trivial way. Following the symplectic classification of simple semi-toric systems achieved by Pelayo and Vũ Ngọc, one of the five symplectic invariants of an integrable semi-toric system is a Taylor series associated to each focus-focus point. The first term of this series is already known for some semi-toric integrable systems and in the case of the spherical pendulum, a system with no proper momentum map, the higher order terms of the series have been computed by Dullin. In our case, the Hamiltonian is expanded near the unstable focus-focus point using explicit expressions for the action and a modified Birkhoff normal form. To our knowledge this is the first explicit calculation of the higher order terms of the Taylor series invariant near a focus-focus point of a semi-toric invariant with a proper momentum map.

Integrable Hénon–Heiles systems on curved spaces: The Sawada–Kotera case

Alfonso Blasco, Universidad de Valladolid
E-mail address: ablasco@ubu.es.

Amongst the three (Liouville) integrable Hénon–Heiles systems on the two-dimensional Euclidean plane [1], we consider the Sawada–Kotera Hamiltonian given by

$$\mathcal{H} = \frac{1}{2}(p_1^2 + p_2^2) + \delta(q_1^2 + q_2^2) + \gamma \left(q_1^2 q_2 + \frac{1}{3} q_2^3 \right),$$

where δ and γ are two arbitrary real parameters. This system corresponds to the superposition of an isotropic oscillator (whenever $\delta > 0$) and a cubic potential. It is well-known that the corresponding integral of motion $\mathcal{I}$ is quadratic in the momenta and that $\mathcal{H}$ is separable in rotated Cartesian coordinates: $u = q_1 + q_2$ and $v = q_1 - q_2$.

In this contribution, we present the constant curvature analogue of $\mathcal{H}$, denoted $\mathcal{H}_\kappa$, on the two-dimensional sphere and the hyperbolic plane [2], where the real parameter κ is just the (Gaussian) curvature of the underlying Riemannian space. In our approach, the curvature κ can be regarded as a deformation/contraction parameter [3], in such a manner that the Hénon–Heiles Euclidean system $\mathcal{H}$ is recovered under the flat limit $\kappa \to 0$. In particular, we explicitly obtain the curved constant of motion $\mathcal{I}_\kappa$ (which remains as a quadratic one in the momenta) and, furthermore, we solve the Hamilton–Jacobi separability of the curved system on an appropriate curved counterpart of the rotated Cartesian coordinates (u, v).

We stress that the separability of $\mathcal{H}_\kappa$ leads to a new series of integrable (curved) homogeneous potentials, similarly to what happens with the Ramani–Dorizzi–Grammaticos homogeneous potentials [4] related to the Hénon–Heiles KdV Hamiltonian, which have already been studied for the curved case in [3, 5].

This is a joint work with Ángel Ballesteros, Francisco J. Herranz, and Mariano Santander.

References

[1] A.P. Fordy. *The Hénon–Heiles system revisited.* Physica D 52 (1991), 204.
[2] A. Ballesteros, A. Blasco, F.J. Herranz, and M. Santander. *Integrable Hénon-Heiles systems on curved spaces: The Sawada-Kotera case.* (2017), submitted.
[3] A. Ballesteros, A. Blasco, F.J. Herranz, and F. Musso. *An integrable Hénon-Heiles system on the sphere and the hyperbolic plane.* Nonlinearity 28 (2015), 3789.
[4] A. Ramani, B. Dorizzi, and B. Grammaticos. *Painlevé conjecture revisited.* Physical Review Letters 49 (1982), 1539.
[5] A. Ballesteros, A. Blasco, and F.J. Herranz. *A curved Hénon–Heiles system and its integrable perturbations.* Journal of Physics: Conference Series 597 (2015), 012013.

***N*-body and *N*-vortex dynamics on surfaces: curvature and topological contributions**

Stefanella Boatto, Universidade Federal de Rio de Janeiro
E-mail address: boatto.stefanella@gmail.com.

One of the today's challenges is the formulation of the N-body and N-vortex dynamics on Riemann surfaces. In this talk we show how the two problems are strongly related one another when looking at them from the point of view of the intrinsic geometry of the surface where the dynamics takes place. Given a surface M of metric g, the distribution of matter S on M, we deduce the dynamics of the masses and some of its properties.

Among other things, we find that in the plane the two masses problem does not obeys to the known Kepler laws. Moreover, Newton's Laws are not longer verified on closed surfaces with variable curvature. For masses on an infinite cylinder we are able to observe topological effects in the dynamics

This is a joint work with David Dritschel, Gladston Duarte, Teresa J. Stuchi, and Renata Loiola.

References

[1] S. Boatto, D.G. Dritschel. *N-body dynamics on surfaces: the role of the curvature's surface and boundaries in particle dynamics.* In preparation.

[2] S. Boatto, G. Duarte, R. Loiola, and T. Stuchi. *N-body and N-vortex dynamics on surfaces: curvature and topological contributions.* In preparation

[3] S. Boatto, R. Schaefer, and D. Dritschel. *N-body dynamics on closed surfaces: the axioms of mechanics.* Proceedings of Royal Society A 472 (2016), 20160020.

[4] D. Dritschel and S. Boatto. *The motion of point vortices on closed surfaces.* Proceedings of Royal Society A 471 (2015), 20140890.

[5] G. Durate. *Two body dynamics over the surface of an infinite cylinder.* Tese de Mestrado em Matemática Aplicada, UFRJ (2016).

[6] R. Loiola. *Vortex and mass dynamics over hyperbolic surfaces.* Tese de Mestrado Matemática, UFRJ (2016).

[7] R. Schaefer. *A dinâmica de N corpos sobre superfícies: um exemplo para 2 corpos.* Tese de Mestrado em Matemática, UFRJ (2014).

Multiplicative quiver varieties and generalised Ruijsenaars-Schneider models

Maxime Fairon, University of Leeds
E-mail address: mmmfai@leeds.ac.uk.

It is well known that the Calogero-Moser system can be obtained by performing a Hamiltonian reduction on the cotangent bundle for $\mathfrak{gl}_n$. One can also view this reduction as performed on the cotangent bundle for the space of representations of a one-loop quiver. It has been observed by Van den Bergh [VdB1] that large part of this Hamiltonian picture, for any quiver, can be understood using the formalism of double Poisson brackets. Furthermore, he showed that a space of representations of an arbitrary quiver also admits a natural quasi-Hamiltonian structure. Thus, one can perform a quasi-Hamiltonian reduction and obtain in this way the so-called multiplicative quiver varieties.

We explain how the Ruijsenaars-Schneider model and its variants can be derived in this context, starting from a very simple quiver. Moreover, we show that the integrability of these systems is already present at the noncommutative algebra level. We also consider a more general case of a cyclic quiver and find new integrable systems of Ruijsenaars–Schneider type.

This is a joint work with Oleg Chalykh (Leeds).

References

[VdB1] M. Van den Bergh. *Double Poisson algebras.* Trans. Amer. Math. Soc. 360(11) (2008), 5711–5769.

Persistence of relative equilibria in Hamiltonian systems under explicit symmetry breaking

Marine Fontaine, The University of Manchester
E-mail address: marine.fontaine@postgrad.manchester.ac.uk.

Explicit symmetry breaking occurs when a dynamical system having a certain group of symmetries is perturbed in a way that the perturbation conserves only some symmetries of the original system. Simple examples include perturbing the rotational motion of small molecules by adding a weak magnetic field or perturbing a symmetric heavy top to one that is not symmetric. For non-Hamiltonian systems, Lauterbach and Roberts [1] looked at what happened to group orbits of equilibria under a perturbation that breaks the symmetry of the system. Some equilibria persist and are connected by heteroclinic cycles. Their result relies on the assumption that the group orbit is normally hyperbolic. In Hamiltonian systems, the conservation of momentum guaranteed by Noether Theorem prevents this assumption from holding. We give a lower bound for the number of orbits of equilibria and orbits of relative equilibria that will persist after a small perturbation, subject to some non-degeneracy conditions and a regularity condition that depends on the geometry.

This is a joint work with James Montaldi.

References

[1] R. Lauterbach and M. Roberts. *Heteroclinic cycles in dynamical systems with broken spherical symmetry.* J. Differential Equations, 100(1) (1992), 22–48.

A Hamilton-Jacobi Theory for general dynamical systems and integrability by quadratures in symplectic and Poisson manifolds

Sergio Grillo, Instituto Balseiro
E-mail address: sergrill@gmail.com.

In this talk we develope, in a geometric framework, a Hamilton-Jacobi Theory for general dynamical systems. Such a theory contains the classical theory for Hamiltonian systems on a cotangent bundle and recent developments in the framework of general symplectic, Poisson and almost-Poisson manifolds (including some approaches to a Hamilton-Jacobi theory for nonholonomic systems). Given a dynamical system, we show that every complete solution of its related Hamilton-Jacobi Equation (HJE) gives rise to a set of first integrals, and vice versa. From that, and in the context of symplectic and Poisson manifolds, a deep connection between the HJE and the (non)commutative integrability notion, and consequently the integrability by quadratures, is stablished. Moreover, in the same context, we find conditions on the complete solutions of the HJE that also ensures integrability by quadratures, but they are weaker than those related to the (non)commutative integrability. Examples are developed in order to illustrate the theoretical results.

Periodic points, Lie symmetries and non-integrability of planar maps
Víctor Mañosa, Universitat Politècnica de Catalunya
E-mail address: victor.manosa@upc.edu.

We consider the problem of characterizing the local $\mathcal{C}^m$-non-integrability, for some $m \in \mathbb{N}$, near elliptic fixed points of smooth planar measure preserving maps. The main result is:

Theorem 1. Let F be a $\mathcal{C}^{2n+2}$-planar map defined on an open set $\mathcal{U} \subseteq \mathbb{R}^2$ with an elliptic fixed point p, not $(2n+1)$-resonant, and such that its first non-vanishing Birkhoff constant is $B_n = i\, b_n$, for some $0 < n \in \mathbb{N}$ and $b_n \in \mathbb{R} \setminus \{0\}$. Moreover, assume that F is a measure preserving map with a non-vanishing density $\nu \in \mathcal{C}^{2n+3}$. If, for an unbounded sequence of natural numbers $\{N_k\}_k$, F has finitely many N_k-periodic points in $\mathcal{U}$ then it is not $\mathcal{C}^{2n+4}$-locally integrable at p.

The proof of our criterion relates the integrability with the existence of some *Lie Symmetries* and with the finiteness of periodic periodic points of the maps. A Lie Symmetry of a map is an associated vector field that gives interesting dynamical information of the map, specially in the integrable case.

Theorem 1 and the rest of results of this work can be applied to study the local integrability of a variety of maps, obtaining in particular the non-integrability of the Cohen's one

$$F(x,y) = \left(y, -x + \sqrt{y^2+1}\right).$$

Theorem 2. The Cohen map is not $\mathcal{C}^6$-locally integrable at its fixed point $\left(\sqrt{3}/3, \sqrt{3}/3\right)$.

This is a joint work with Anna Cima, Armengol Gasull.

Acknowledgements. Supported by Ministry of Economy, Industry and Competitiveness of the Spanish Government through grants MINECO/FEDER MTM2016-77278-P (first and second authors) and DPI2016-77407-P (AEI/FEDER, UE, third author).

Complete equivalence set and mutually unbiased bases in finite Hilbert space

Semiu O. Oladejo, Gombe State University
E-mail address: abdsemiu@yahoo.com.

Equivalency between set of point in pairs of lines in $\mathbb{Z}_2^d$ is studied. Lines in $\mathbb{Z}_2^d$ are produced by acting symplectic operator on an arbitrary line in $\mathbb{Z}_2^d$, a new set of points is produced as a result which tallies with another set of point another line in the same geometry $\mathbb{Z}_2^d$. A connection is created between lines in $\mathbb{Z}_2^d$ and mutually unbiased bases in finite Hilbert space $\mathbb{H}_d$.

Differential Galois theory and Darboux transformations for integrable systems

Raquel Sánchez-Cauce, Universidad Autónoma de Madrid
E-mail address: raquel.sanchezc@uam.es.

We study how the Galois groups of the linear differential systems of AKNS type are transformed under Darboux transformations. In [1], we compute the differential Galois group for Schrödinger operator $-\partial^2 + u - \lambda$ for solitonic solutions u of KdV equation. Now, we present the calculus of differential Galois group for such operator for solitonic solutions u of Modified KdV equation

$$u_t + 6u^2 u_x + u_{xxx} = 0.$$

The AKNS systems techniques and the Picard-Vessiot Theory for integrable systems (as appears in Appendix D of [2]) turn out to be essential in our approach to the problem and allow us to state our main result that says that the Galois group of the transformed system is isomorphic to a subgroup of the Galois group of the initial system, proving that, by Darboux transformations, we obtain systems at least as integrable as the initial system. In particular, if the initial system is integrable, the transformed system is integrable too, in complete agreement with Darboux ideas.

This is a joint work with Sonia Jiménez, Juan J. Morales-Ruiz and María-Ángeles Zurro.

References

[1] S. Jiménez, J.J. Morales-Ruiz, R. Sánchez-Cauce, and M.A. Zurro. *Differential Galois theory and Darboux transformations for integrable systems.* Journal of Geometry and Physics 115 (2017), 75–88.

[2] M. van der Put and M.F. Singer. *Galois theory of linear differential equations*, in: Grundlehren der mathematischen Wissenschaften, Volume 328, Springer Verlag, (2003).

Integrability of the dynamics of a heavy ball in a rotating cup

Nicola Sansonetto, University of Verona
E-mail address: nicola.sansonetto@gmail.com.

Nonholonomic systems with constraints that are affine functions of the velocities do not typically conserve energy, nevertheless there might exists modifications of it, called moving energies, that are first integrals of the systems.

In this talk we will discuss the conservation of the energy for nonholonomic systems with affine constraints and investigate the existence of moving energies if the energy is not preserved. Then we apply the obtain results to investigate the integrability of the system of a heavy homogeneous sphere that rolls without sliding inside a convex surface of revolution in uniform rotation around its vertical figure axis.

References

[1] P. Balseiro and N. Sansonetto. *A geometric characterization of certain first integrals for nonholonomic systems with symmetries.* SIGMA 12 (2016), 14 pages.

[2] F. Fassò, A. Giacobbe, and N. Sansonetto. *Gauge conservation laws and the momentum equation in nonholonomic mechanics.* Rep. Math. Phys. 62 (2008), no. 3, 345–367.

[3] F. Fassò, L. García-Naranjo, and N. Sansonetto. *Moving energies as first integrals of non-holonomic systems with affine constraints.* `arXiv:16111.08626v1[mathDS]`.

[4] F. Fassò and N. Sansonetto. *Conservation of 'moving energy' in nonholonomic aystems with affine constraints and integrability of spheres on rotating surfaces.* J. Non. Sci. 26 (2016), 519–544.

[5] F. Fassò and N. Sansonetto. *Conservation of energy and momenta in nonholonomic systems with affine constraints.* Regul. Chaotic Dyn. 2 (2015), 449–462.

Momentum polytopes and dynamics of the action of $SU(3)$ on products of $\mathbb{C}P^2$

Amna Shaddad, University of Manchester
E-mail address: amna.shaddad@manchester.ac.uk.

The torus action and the resulting convex moment polytope have been studied in detail but little has been done on the combined fixed point set image of the momentum map of the torus and non-torus action. This has been achieved by describing a particular action with weighted symplectic form: we have shown that the momentum polytopes of an action with 'weighted symplectic form' fall into different categories separated by the ratios between the weightings. The momentum polytopes of the action of SU(3) on two copies of CP^2 fall into six different categories. For the SU(3) action on three copies of CP^2 there are nine different distinct polytopes. The polytopes of these actions are separated by transitional polytopes. The poster will show the classification and dynamics of momentum polytopes of the SU(3) action on points in the complex projective plane.

Geodesic equivalence of metrics on Lie groups
Tijana Šukilović, University of Belgrade
E-mail address: tijana@matf.bg.ac.rs.

Since the time of Herman Weyl, geodesically equivalent metrics were actively discussed in the realm of general relativity. The context of general relativity poses the following restrictions: the dimension is 4, the metrics are pseudo-Riemannian of Lorentz signature $(-,+,+,+)$. Sometimes the metrics satisfy additional assumptions such that one or both metrics are Ricci-flat (the vacuum $R_{ij}=0$), or Einstein ($R_{ij}=\frac{R}{4}g_{ij}$), or, more generally, satisfy the Einstein equation $R_{ij}-\frac{R}{2}g_{ij}=T_{ij}$ with "physically interesting" stress-energy tensor T_{ij}. We also point out that geodesic equivalence of metrics is closely related to theory of integrable Hamiltonian systems.

Although, in general, for geodesically equivalent metrics g and $\bar{g}$, their corresponding Levi-Civita connections ∇ and $\bar{\nabla}$ may still be expected to differ, as it turns out in many interesting situations they are necessarily equal.

V.S. Matveev gave an algorithm how to obtain a list of pairs of all possible geodesically equivalent metrics. G.S. Hall and D.P. Lonie have also constructed, using different approach, geodesically equivalent 4-dimensional metrics $g,\bar{g}$ of Lorentz signature.

So far there are no results in this area in the case of Lie groups.

Semilocal invariants of multi-pinched focus-focus singularities
Xiudi Tang, University of California at San Diego
E-mail address: xdtang@ucsd.edu.

We classify, up to symplectomorphisms, a neighborhood of a focus-focus singularity of an integrable system with $k\in\mathbb{N}$ focus-focus critical points on the same fiber. The case $k=1$ was solved by San Vũ Ngọc in 2003, who also made a conjecture for general k. The goal of this paper is to prove this conjecture.

This is the first of a series of papers in which we study focus-focus singularities of integrable systems and generalizations of Vũ Ngọc's conjecture.

5. List of Participants

Anton Alekseev	Université de Genève
Jaume Alonso	Antwerp University
Philip Arathoon	University of Manchester
Angel Ballesteros	Universidad de Burgos
Romero Barbieri	Pontificia Universidade Católica do Rio de Janeiro
Michael Bialy	Tel Aviv University
Alfonso Blasco	Universidad de Burgos
Stefanella Boatto	Universidade Federal do Rio de Janeiro
Sergey Bolotin	University of Wisconsin - Madison
Alexey Bolsinov	Loughborough University
Damien Bouloc	Université Paul Sabatier
Roisin Braddell	Trinity College
Harry Braden	University of Edinburgh
Roger Casals	Massachusetts Institute of Technology
Oleg Chalykh	University of Leeds
Yunhyung Cho	Sungkyunkwan University
Pantelis Damianou	University of Cyprus
Amadeu Delshams	Universitat Politècnica de Catalunya
Renata Dos Santos	IMPA - Intituto de Matemática Pura e Aplicada
Gladston Duarte	Universitat de Barcelona
Konstantinos Efstathiou	University of Groningen
Viktor Enolski	National University of Kyiv-Mohyla Academy
Maxime Fairon	University of Leeds
Gerard Farré	Kungliga Tekniska Högskolan
László Fehér	Wigner Research Centre for Physics, Budapest
Marine Fontaine	University of Manchester
Allan Fordy	University of Leeds
Patrick Foulon	CIRM Lumini
Carlos García	Universidad Nacional Autónoma de México
Otávio M. L. Gomide	Universidade Estadual de Campinas
Tamás F. Görbe	University of Szeged
Sergei Gukov	California Institut of Technology
Sonja Hohloch	Antwerp University
Shinsuke Iwao	Tokai Univesity, Tokyo
Anton Izosimov	University of Toronto
Charles Jaffe	West Virginia University

Kai Jiang	Peking Univesity
Božidar Jovanovic	Mathematical Institute of the SANU, Belgrad
Vadim Kaloshin	University of Maryland
Fariba Khoshnasib	University of Texas at Dallas
Vladislav Kibkalo	Moscow State University
Andreas Knauf	Friedrich-Alexander-Universität-Erlangen
Andrei Konyaev	Moscow State University
Sasa Kresic-Juric	Sveucilište u Splitu
Boris Kruglikov	University of Tromso
Hieronim Kubica	AGH University of Science and Technology
Elena Kudryavtseva	Moscow State University
Oswaldo José Larreal	University of Zulia
Yohann Le Floch	Université de Strasbourg
Carlos Leon	Université de Poitiers
Andrzej Maciejewski	University of Zielona Gora
Leyli Mammadova	Katholieke Universiteit Leuven
Giovanni Manno	Politecnico di Torino
Víctor Mañosa	Universitat Politècnica de Catalunya
David Martínez	PUC do Rio de Janeiro
Nikolay Martynchuk	University of Groningen
Phetogo Masemola	University of the Witwatersrand
Vladimir Matveev	Friedrich-Schiller Universität Jena
Thomas Mettler	Goethe Universität Frankfurt
Eva Miranda	Universitat Politècnica de Catalunya
Philippe Monnier	Université Paul Sabatier
Stanislav Nikolaienko	Lomonosov Moscow State University
Cédric Oms	Universitat Politècnica de Catalunya
Andrey Oshemkov	Lomonosov Moscow State University
Shin-ichi Otani	Kanto Gakuin University
Joseph Palmer	Rutgers University
Andriy Panasyuk	University of Warmia and Mazury in Olsztyn
Jose Luis Pérez	ICMAT/CSIC Madrid
Andres Perico	University of California at Santa Cruz
Francisco Presas	Instituto de Ciencias Matemáticas
Emma Previato	Boston University
Maria Przybylska	University of Zielona Gora
Sanjay Ramassamy	École Normale Supérieure de Lyon
Witold Respondek	INSA de Rouen

Raquel Sánchez	Universidad Autónoma de Madrid
Rodrigo Schaefer	Universitat Politècnica de Catalunya
Amna Shaddad	University of Manchester
Vsevolod Shevchishin	University of Warmia and Mazury in Olsztyn
Taras Skrypnyk	Università degli Studi di Milano-Bicocca
Christos Sourdis	Christos Sourdis Sourdis
Esmeralda Sousa	Instituto Superior Técnico Lisboa
Tijana Šukilović	University of Belgrade
Yuri B. Suris	Technische Universität Berlin
Wojciech Szuminski	University of Zielona Gora
Sergei Tabachnikov	The Pennsylvania State University
Xiudi Tang	University of California at San Diego
Daisuke Tarama	Ritsumeikan University
Kentaro Tomoda	Kobe University
Dmitry Treschev	Lomonosov Moscow State University
Mikhail Tuzhilin	Lomonosov Moscow State University
Galliano Valent	Laboratoire de Physique Mathematique de Provence
Pol Vanhaecke	Université de Poitiers
Mats Vermeeren	Technische Universität Berlin
Andreas Vollmer	Politecnico di Torino
Christophe Wacheux	École Polytechnique Fédérale de Lausanne
Thomas Waters	University of Portsmouth
Jonathan Weitsman	Northeastern University
Jiaqi Yang	Georgia Institute of Technology
Lei Zhang	Georgia Institute of Technology