Extreme values of the Riemann zeta function

We prove that for every $c < 1/\sqrt{2}$ there exists arbitrarily large T with

$$|\zeta(1/2 + iT)| > \exp\left(c\sqrt{\log T \log\log\log T / \log\log T}\right).$$

This improves classical results by Montgomery, Balasubramanian-Ramachandra, and Soundararajan. Our proof uses Soundararajan's resonance method, multiplicative functions, and a certain large greatest common divisor sum.