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A Fixed-Point Approach to Barycenters in Wasserstein Space.

Abstract: Let $\mathcal{P}_{2,ac}$ be the set of Borel probabilities on $\mathbb{R}^d$ with finite second moment and absolutely continuous with respect to Lebesgue measure. We consider the problem of finding the barycenter (or Fréchet mean) of a finite set of probabilities $\nu_1, \ldots, \nu_k \in \mathcal{P}_{2,ac}$ with respect to the L_2-Wasserstein metric. For this task we introduce an operator on $\mathcal{P}_{2,ac}$ related to the optimal transport maps pushing forward any $\mu \in \mathcal{P}_{2,ac}$ to $\nu_1, \ldots, \nu_k$. Under very general conditions we prove that the barycenter must be a fixed point for this operator and introduce an iterative procedure which consistently approximates the barycenter. The procedure allows effective computation of barycenters in any location-scatter family, including the Gaussian case. In such cases the barycenter must belong to the family, thus it is characterized by its mean and covariance matrix. While its mean is just the weighted mean of the means of the probabilities, the covariance matrix is characterized in terms of their covariance matrices $\Sigma_1, \ldots, \Sigma_k$ through a nonlinear matrix equation. The performance of the iterative procedure in this case is illustrated through numerical simulations, which show fast convergence towards the barycenter.