

# Adaptation to lowest density regions with application to support recovery

## Abstract:

A scheme for locally adaptive bandwidth selection is proposed which sensitively shrinks the bandwidth of a kernel estimator at lowest density regions such as the support boundary which are unknown to the statistician. In case of a Hölder continuous density, this locally minimax-optimal bandwidth is shown to be smaller than the usual rate, even in case of homogeneous smoothness. Some new type of risk bound with respect to a density-dependent standardized loss of this estimator is established. This bound is fully nonasymptotic and allows to deduce convergence rates at lowest density regions that can be substantially faster than  $n^{-1/2}n^{1/2}$ . It is complemented by a weighted minimax lower bound which splits into two regimes depending on the value of the density. The new estimator adapts into the second regime, and it is shown that simultaneous adaptation into the fastest regime is not possible in principle as long as the Hölder exponent is unknown. Consequences on plug-in rules for support recovery are worked out in detail. In contrast to those with classical density estimators, the plug-in rules based on the new construction are minimax-optimal, up to some logarithmic factor.