AXIOMATIC HOMOTOPY THEORY VIA GROTHENDIECK DERIVATORS

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The theory of derivators goes back to Alex Heller, Alexander Grothendieck, Jens Franke, and others, and it addresses the following slogan: 'The passage from an abelian category to its derived category results in a loss of information.' In particular, the calculus of derived limits and derived colimits is not visible to the derived category *alone*. For example, the non-functoriality of the cone construction on derived categories is a reminiscent of this fact. This and other problems can be avoided if one considers simultaneously derived categories of various diagram categories. Derivators axiomatize this idea, thereby providing a minimal extension of the classical derived categories to a categorical framework encoding the rich and non-trivial calculus of derived (co)limits.

More generally, derivators are an approach to abstract homotopy theory, focusing on the rich and interesting calculus of homotopy limits and homotopy Kan extensions, a calculus in which various areas of the IRTACTA program meet. For example, specializing to stable derivators (enhancements of triangulated categories), this calculus has applications in abstract representation theory. In particular, it allows to extend certain classical derived equivalences from representations over fields to more general contexts like representations over rings, schemes, dgas, or ring spectra.

In these lectures we give an introduction to the basic theory of derivators, discuss the relation to (higher) triangulated categories, and mention first applications to representation theory.

Lectures:

(1) Motivation, homotopy limits and colimits, axioms of derivators, outlook towards canonical triangulations.
(2) Canonical triangulations, derivators as enhancements of triangulated categories. Examples of derivators: derivators of rings, spaces, and spectra.
(3) Towards abstract representation theory: strong stable equivalences, reflection functors
(4) Abstract representation theory of A_n-quivers, higher triangulations.

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