

Advanced Course
on
Recent Trends of Combinatorics
in the Mathematical Context

Notes of the Course

September 13 to 23, 2005
Centre de Recerca Matemàtica
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Béla Bollobás

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Foreword

This notes cover the contents of two sets of lectures given at the Advanced Course ‘Recent Trends in Combinatorics in the Mathematical Context’ given at the CRM in September 2005.

The title of the course intends to reflect the increasing connections between combinatorics and other established fields of Mathematics. The topics selected in this occasion under this broad denomination were ‘Combinatorial Probability’, a section delivered by Prof. Béla Bollobás, and ‘Combinatorics of Morphisms’ given by Prof. Jaroslav Nešetřil. Both areas have been the subject of intense and fruitful research activity in the last decades and the two aforementioned authors have been two of the most significant leaders in each of them.

The notes give both an introduction to the classical concepts and references to recent developments in each area.

One of the key results described in the Combinatorial Probability part was the recently obtained Reimer’s inequality which gives an extension to non monotone classes of events of the more classical van den Berg-Kesten inequality.

Highlight results from the second part on Combinatorics of Morphisms include the universality of categories of graphs, the Nešetřil-Shelah theorem on antichains, and the constructability of chromatic graph classes (Hajos’s theorem).

The notes in each of the two parts are extracted from preliminary versions of two forthcoming books by the same authors and titles. We are most grateful to Prof. Bollobás and to Prof. Nešetřil for kindly providing this material for the present edition.

The course was attended by some 60 students coming from more than 20 countries.

We are also extremely grateful for the financial support given by the European Union through the programme Marie Curie Conferences and Training Course, and to the European Mathematical Society which hosted the course within its programme of Summer Schools.

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Oriol Serra
Coordinator
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The notes contained in this booklet were printed directly from files supplied by the authors before the course.

COMBINATORICS OF MORPHISMS

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