Course 1: p-adic Galois representations and global Galois deformations

a) L. Berger: p-adic Galois representations

The group G_Q

Decomposition subgroups

Ramification

ell ≠ p -adic representations

ell = p -adic representations

examples

F_p - linear representations

cohomology of Galois representations

rings of periods

crys, sst and dR representations

p-adic Hodge theory

the fundamental exact sequence

Bloch-Kato's maps

dimensions of the H^1_e,f,g

(phi,Gamma)-modules

reciprocity theorems

Trianguline representations

Wach modules

mod p reduction

b) G. Böckle: Deformations of Galois representations

Lecture I: Deformations of representations of pro-finite groups

Deformation Functors

Tangent Spaces

Presentations of universal rings via generators and relations

Groupoids over categories

Lecture II: Quotients by group actions and Pseudo-Representations

Quotients by group actions

Pseudo-representations

Deformations of pseudo-representations

Comparison with Deformations of representations

Lecture III: p-adic deformations away from p and ordinary representations

The generic fiber of a deformation functor

(The very) basics of Weil-Deligne representation

Deformations of mod p representations for \ell ≠ p

Their generic fibers

Ordinary deformation functors

Lecture IV: Flat deformations

Flat deformation functors

Smoothness of the generic fiber (via weakly admissible modules)

Smoothness when e=1 (via Fontaine-Laffaille)

Outlook

Lecture V: Presenting Global deformation rings over local ones

Tangent spaces

Relative presentations

Geometric deformation conditions

The bound dim R_\geom >= 1

Course 2: J-P. Wintenberger: Modularity Lifting Theorems and Potential Modularity

1)Introduction.

- Galois representations attached to automorphic forms

- Modularity and potential modularity of Galois representations

2) Potential modularity theorem

- Statement and the plan of the proof

- existence of abelian varieties

- end of the proof.

3) Modularity lifting theorem

- Automorphic representations for quaternion algebras.

Morphism $R\rightarrow T$

- Auxiliary primes ; freeness of space of modular forms

- Galois cohomology

- Patching argument

Bibliography:

For introduction:

F. Diamond and J. Shurman.

A first course in Modular Forms.

Graduate Texts in Mathematics, 228, 2005.

J.-M. Fontaine and Y. Ouyang

Theory of p-adic Galois Representations

Jean-PIerre Serre

Abelian l-adic representations and elliptic curves,

J.-M. Fontaine and B. Mazur.

Geometric Galois representations.

Elliptic curves, modular forms, \& Fermat's last theorem

(Hong Kong, 1993), Ser. Number Theory, I, pages 41--78. Internat. Press,

Cambridge, MA, 1995.

Richard Taylor.

Galois representations.

Annales de la Facult\'e des Sciences de Toulouse 13 (2004),

For Potential modularity:

Richard Taylor.

Remarks on a conjecture of Fontaine and Mazur.

Inst. Math. Jussieu, 1(1):125--143, 2002.

Richard Taylor.

On the meromorphic continuation of degree two {L}-functions.

Documenta Math. Extra Volume: John H. Coates' Sixtieth Birthday (2006) 729--779.

For modularity theorems:

H. Darmon, F. Diamond, R. Taylor.

Fermat's last theorem.

Elliptic curves, modular forms \& Fermat's last theorem

(Hong Kong, 1993), 2-140, Int. Press, Cambridge, MA,1997.

Mark Kisin

Moduli of finite flat group schemes and modularity

Annals of Math. 170(3) (2009), 1085-1180.

Mark Kisin

The Fontaine-Mazur conjecture for GL_2

J.A.M.S 22(3) (2009) 641-690.

A. Wiles.

Modular elliptic curves and Fermat's last theorem.

Ann. of Math. (2), 141(3), 443--551, 1995.

R. Taylor and A. Wiles.

Ring-theoretic properties of certain Hecke algebras.

Ann. of Math. (2), 141(3), 553--572, 1995.

Course 3: F. Diamond: Serre's conjectures and generalizations

1) Introduction - statement of Serre's conjecture

Serre's conjecture, now a Theorem of Khare-Wintenberger, states that every continuous, odd, irreducible two-dimensional representation of G_Q arises from modular forms. We will recall the statement of the conjecture, including Serre's recipe for the minimal level and weight of the modular form.

We will also discuss a refinement of the weight part of the conjecture with a view towards its generalizations.

2) Proof of Serre's conjecture (talks by L. Dieulefait)

--Review of the MLT that will be used (results of Taylor-Wiles, Skinner-Wiles, Diamond, Kisin). Review of results of Böckle on presentations of universal global deformation rings.

--Review of results of Tate, Serre, Fontaine, Schoof and Brumer-Kramer on Galois representations and abelian varieties of small prime conductor.

--Consequences of potential modularity (results of Dieulefait and Khare-Wintenberger): existence of compatible systems, existence of Galois conjugates, and existence of lifts with prescribed properties; proof of the first cases of the Fontaine-Mazur and Serre's conjectures

--Induction on the weight (Khare): proof of the level 1 case. An alternative proof (by Dieulefait) via Galois conjugation.

--Proof of the general case (Khare-Wintenberger): new tools: good-dihedral primes, 2-adic MLT, and more weight induction.

--An alternative proof of the odd level case (Dieulefait): new tool: pseudo Sophie Germain primes and killing ramification (via results of Kisin, Caruso and Schoof)

3) Serre's conjecture for GL_2 over totally real fields

Buzzard-Diamond-Jarvis formulated a generalization of Serre's conjecture to GL_2 over a totally real field in which p is unramified, the main difficulty being the specification of the weight. We will discuss the statement of the conjecture, generalizations of Schein and Gee to the ramified case, and results on the weight part of the conjecture.

In particular, we explain Gee's method via modularity of lifts of prescribed type in the unramified case, and results of Gee-Savitt via weight-cycling in the ramified case.

4) Langlands correspondences mod p

After reviewing some of the framework for Langlands program for GL_2, we describe how Serre's conjecture can be viewed as part of a "mod p Langlands program," with the specification of level and weight being corollaries of a local-global compatibility statement made precise by work of Emerton.

5) Serre's conjecture for GL_n

We will explain the statement of generalizations of Serre's conjecture to the context of automorphic representations of GL_n, following Herzig and Gee-Savitt.

Bibliography:

J.-P. Serre, Sur les representations modulaires de degre 2 de Gal(Qbar/Q), Duke Math. J. 54 (1987), 179--230.

C. Khare, J.-P. Wintenberger, Serre's modularity conjecture I, Invent. Math. 178 (2009), 485--504

K. Buzzard, F. Diamond, F. Jarvis, On Serre's conjecture for mod l Galois representations over totally real fields to appear in Duke Math J. available from http://www.mth.kcl.ac.uk/staff/f_diamond.html

M. Emerton, Local-global compatibility in the p-adic Langlands programme for GL_2/Q http://www.math.northwestern.edu/~emerton/preprints.html

F. Herzig, The weight in a Serre-type conjecture for tame n-dimensional Galois representations, Duke Math. J. 149 (2009), 37-116