Conference on

Geometry and Topology of Foliations

CRM, July 12-16

Program

Monday, July 12

9:00 - 9:30. Registration

9:30 - 10:30. Schweitzer, P., On the structure of homological Reeb components in codimension one foliations

10:30 – 11:00. Coffee break.

11:00 – 12:00. Megniez, G., Haefliger structures of codimension 1 made regular and minimal.

12:00 – 13:00. Miyoshi, S., A construction of a typical foliation on a 3-manifold

13:00 – 15:00. Lunch.

15:00 – 16:00. Bis, A., Denjoy-Sacksteder theory for groups of diffeomorphisms

16:00 – 16:15. Break.

16:15 – 17:15. Czarnecki, M., Umbilical foliations of hyperbolic spaces

17:15 – 18:15. Volk, D., Skew products with interval fiber

Tuesday, July 13

9:30 - 10:30. Mitsumatsu, Y., Turbulization of 2-dimensional foliations on 4-manifolds and tautness

10:30 – 11:00. Coffee break.

11:00 – 12:00. Tsuboi, T., On the uniform perfectness of the group of diffeomorphisms

12:00 – 13:00. Ishida, T., Second cohomology classes of the group of C^1-flat diffeomorphisms of the line

13:00 – 15:00. Lunch.

15:00 – 16:00. Royo Prieto, J.I., Cohomology of S^3-actions and duality

16:00 – 16:15. Break.

16:15 – 17:15. Toeben, D., Equivariant Basic Cohomology of Riemannian Foliations

17:15 – 18:15. Pozo, L., Cohomology of horizontal forms

Wednesday, July 14

9:30 - 10:30. Hurder, S., To be announced

10:30 – 11:00. Coffee break.

11:00 – 12:00. Candel, A., The Higson compactification and algebraic characterization of quasi-isometric metric spaces

12:00 – 13:00. Lozano Rojo, Á., Affability of laminations of polynomial growth

13:00 – 15:00. Lunch.

Afternoon: excursion.

Thursday, July 15

9:30 - 10:30. Kordyukov, Y., Adiabatic spectral asymptotics on foliated manifolds

10:30 – 11:00. Coffee break.

11:00 – 12:00. Pereira, J.V., The characteristic variety of a generic foliation

12:00 – 13:00. Licanic, S., On boundedness of families of holomorphic foliations

13:00 – 15:00. Lunch.

15:00 – 16:00. Chiossi, S., Käehler surfaces and complex homothetic foliations

16:00 – 16:15. Break.

16:15 – 17:15. Dragomir, S., Harmonic maps of foliated Riemannian manifolds

17:15 – 18:15. Walczak, S., Hausdorff leaf spaces for foliations of codimension one

Friday, July 16

9:30 - 10:30. Bolotov, D., Foliations of nonnegative curvature

10:30 – 11:00. Coffee break.

11:00 – 12:00. Langevin, R., Canal foliations of S^3

12:00 – 13:00. Walczak, P., Extrinsic geometric flows

13:00 – 15:00. Lunch.

15:00 – 16:00. Slesar, V., A vanishing result for the $Spin^c$ Dirac operator defined along the leaves of a foliation

16:00 – 16:15. Break.

16:15 – 17:15. Nozawa, H., 5-dimensional K-contact geometry via the Morse theory on moment maps

Abstracts

Andrzej Bis (University of Lodz)

Title: Denjoy-Sacksteder theory for groups of diffeomorphisms

(Joint work with Gilbert Hector)

Abstract: Let G be a finitely generated group of C^2 -difeomorphisms of the circle. If it admits a Cantor set E as minimal set, there exist g \in G and x \in E such that g(x) = x and |g′(x)|< 1. This is the celebrated theorem of Sacksteder for group actions (see [5]) and our goal here is to extend it to suitable finitely generated groups of diffeomorphisms of closed manifolds. To do so, we first introduce the notion of "exceptional minimal sets". A minimal set E for a group G acting on a closed manifold M will be called "exceptional" if it is connected, has empty interior and is such that the open set M\E has infinitely many components all of whose closures are pairwise disjoint. Now it happens that, for any ε > 0, McSwiggen constructed in [4] infinite cyclic groups of $C^{3-\varepsilon}$ -diffeomorphisms of the 2-torus which preserve such an exceptional minimal set but don't admit any non trivial stabilizer. Therefore in order to really extend Sacksteder, we make a double restriction:

i) we assume that the minimal sets E are of "strongly decreasing type" that is the sum of the diameters of the complementary components is bounded ,

ii) the group of diffeomorphisms G are "quasi-conformal" that is the pointwise dilatation of all their elements are uniformly bounded by some constant K.

In this setting, Sacksteder's theorem extends and we provide a proof following quite closely the procedure of proof of the original work of Sacksteder. Indeed Sacksteder's theorem holds under a somewhat weaker differentiability hypothesis and so will it be for our generalization.

References

[1] A. Bis, H. Nakayama, P. Walczak, Locally connected exceptional minimal sets of surface homeomorphisms, Ann. Inst. Fourier, 54(3) (2004), 711-731.

[2] A. Bis, H. Nakayama, P. Walczak, Modelling minimal foliated spaces with positive entropy, Hokkaido Math. J., 36(2) (2007), 283-310.

[3] A. Denjoy, Sur les courbes d ´efinies par les ´equations diff´erentielles la surface du tore, J.Math. Pure Appl., 11 (1932), 333-375.

[4] P. McSwiggen, Diffeomorphisms of the torus with wandering domains, Proc. Amer. Math. Soc., 117 (1993), 1175-1186.

[5] R. Sacksteder, Foliations and pseudogroups, Amer. J. Math., 87 (1965), 79-102.

[6] G. T. Whyburn, Topological characterization of the Sierpi´nski curve, Fund. Math., 45 (1958), 320-324.

Dmitry Bolotov (B. Verkin Institute for Low Temperature Physics)

Title: Foliations of nonnegative curvature

Abstract: The structure of codimension 1 foliations of nonnegative curvature on closed manifolds will be described. A classification of closed orientable three-dimensional manifolds admitting a transversely orientable nonnegatively curved foliation will be presented

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Alberto Candel (CSUN)

Title: The Higson compactification and algebraic characterization of quasi-isometric metric spaces

Abstract: The theme of this talk is to characterize the quasi-isometry type of a proper metric space via certain Banach algebra of functions on it, called the Higson algebra. This algebra gives rise to the Higson compactification of a metric space, and we also show its role in the limit set structure of the leaves of a foliated space. This talk describes work in collaboration with J. A. Alvarez Lopez.

Simon Chiossi (Politecnico di Torino)

Title: Käehler surfaces and complex homothetic foliations

Abstract: Complex Hermitian surfaces that admit an oppositely-oriented orthogonal K□ahler structure are called Käehler-Hermitian surfaces. There is a relationship between the latter and holomorphic, conformal foliations on Käehler 4-manifolds that I will explain from the viewpoint of G-structures.

Calabi's construction of Käehler metrics on Hermitian line bundles over Riemann surfaces will be adapted to show that any Käehler surface equipped with a complex homothetic foliation is locally obtained by a suitable twist of a Calabi-type Käehler structure.

Maciej Czarnecki (Uniwersytet Lodzki, Poland)

Title: Umbilical foliations of hyperbolic spaces

Abstract: According to an easy classification of totally umbilical complete hypersurfaces in the hyperbolic space I will describe properties of totally umbilical codimension 1 foliations of H^n. In particular, I will show formulae for an orthogonal transversal.

Sorin Dragomir (Universita' degli Studi della Basilicata, Italy)

Title: Harmonic maps of foliated Riemannian manifolds

Abstract: In this talk we present the development of the theory of transversally harmonic maps (cf. [3] and [4]-[5]) from foliated Riemannian manifolds (*M,F, g*) i.e. smooth critical points $\phi : M \to N$ of the energy functional $E_T(\phi)$ with respect to variations through foliated maps. In particular we discuss transversally harmonic morphisms i.e. smooth foliated maps preserving the basic Laplace equation $\Delta_B u = 0$. We show that CR maps of compact Sasakian manifolds preserving the Reeb flows are weakly stable transversally harmonic maps. We study transversally harmonic maps into spheres and give foliated analogs to B. Solomon's (cf. [6]) results.

References

[1] E. Barletta & S. Dragomir, *On transversally holomorphic maps of Kählerian foliations*, Acta Applicandae Mathematicae, 54(1998), 121-134.

[2] A. El Kacimi-Alaoui, *Operateurs transversalement elliptiques sur un feuilletage riemannien et applications*, Compositio Mathematica, (1) 73 (1990), 57-106.

[3] A. El Kacimi Alaoui & E. Gallego Gomez, *Foliated harmonic maps*, Illinois J. Math., 40 (1996), 115-122.

[4] J.J. Konderak & R.A. Wolak, *Transversally harmonic maps between manifolds with Riemannian foliations*, Quart. J. Math., 54(2003), 335-354.

[5] J.J. Konderak & R.A. Wolak, *Some remarks on transversally harmonic maps*, Glasgow J. Math., (1) 50 (2008), 1-16.

[6] B. Solomon, *Harmonic maps to spheres*, J. Differential Geometry, 21(1985), 151-162.

A. El Kacimi (Université de Valenciennes)

Title: Guichard and Mittag-Leffler theorems for a complex simple foliation

Abstract: The goal of this lecture is to sketch a proof of the following result: Let (M, F) be a complex simple foliation whose leaves are simply connected non-compact Riemann surfaces and γ an automorphism of F which fixes each leaf and acts on it freely and properly. Then, the vector space $H_F(M)$ of leafwise holomorphic functions is not reduced to functions constant on the leaves and for any g\in $H_F(M)$, there exists h\in $H_F(M)$ such that h-h\circ γ=g. (This is a generalization of Guichard's theorem proved in 1886.) From this proof we derive a foliated version of Mittag-Leffler Theorem.

Steven Hurder (University of Illinois at Chicago)

Title: To be announced.

Tomohiko Ishida (The University of Tokyo)

Title: Second cohomology classes of the group of C^1-flat diffeomorphisms of the line

Abstract: We denote by $\mathrm{Diff}_0(\mathbf{R})$ the group of orientation-preserving C^\infty-diffeomorphisms of **R** which fix the origin. For $k \geq 1$, we denote by $\mathrm{Diff}_k(\mathbf{R})$ the subgroup of $\mathrm{Diff}_0(\mathbf{R})$ consisting of elements which are C^k-flat to the identity at the origin. For $l \geq k$, let α_l's be the 1-cochains of $\mathrm{Diff}_k(\mathbf{R})$ defined by $\alpha_l(f) = d^l/dx^l\, f(0)$ for $f \in \mathrm{Diff}_k(\mathbf{R})$. They are used in [2] to define 2-cocycles γ^2_- and γ^2_+ of the group $\mathrm{Diff}_1(\mathbf{R})$. Our main theorem is the following.

Theorem ([2])**.** *Let* $\gamma^2 : H_2(\mathrm{Diff}_1(\mathbf{R});\mathbf{Z}) \to \mathbf{R}^2$ *be the homomorphism defined by*

$$\gamma^2(\xi) = (\gamma^2_-(\xi),\ \gamma^2_+(\xi)).$$

Then γ^2 *is surjective.*

References:

1. K. Fukui, *Homologies of the group* $\mathrm{Diff}_1(\mathbf{R}^n,0)$ *and its subgroups*, J. Math. Kyoto Univ. **20** (1980), 475–487.

2. T. Ishida, *Second cohomology classes of the group of C^1-flat diffeomorphiosms of the line*, Preprint series UTMS 2010-4.

Yuri A. Kordyukov (Institute of Mathematics RAS, Ufa, Russia)

Title: Adiabatic spectral asymptotics on foliated manifolds

Abstract: In this talk we are going to discuss some recent results concerning the asymptotic behavior of the eigenvalue distribution function of the Laplace operator on a compact Riemannian foliated manifold when the metric on the ambient manifold is blown up in directions normal to the leaves (in the adiabatic limit). In particular we will address the noncommutative Weyl formula and related problems on the distribution of integer points.

Rémi Langevin (Université de Bourgogne)

Title: Canal foliations of S^3.

Abstract: The sphere S^3 does not admit foliations (without singularities) by (round) spheres. Neither does it admit (non-singular) foliations by Dupin cyclides. Relaxing a little more the geometry of the leaves, we found a not too rich family of foliations of S^3: the foliations by canal surfaces, that we can describe completely. A main feature of these foliations is the existence of at least one leaf which is a Dupin cyclide.

Sergio Licanic (Universidade Federal Fluminense, Rio de Janeiro, Brazil)

Title: On boundedness of families of holomorphic foliations

Abstract: We get boundedness for certain families of foliations. This is attained after proving a kind of foliated Szpiro inequality.

Álvaro Lozano Rojo (Univerasidad País Vasco)

Title: Affability of laminations of polynomial growth
(Joint work with P. González Sequeiros)

Abstract: Affability was first introduced by O. Bratteli in the study of C*-algebras. AF-algebras are inductive limits of finite dimensional algebras. They are sufficiently general to contain interesting examples and they have enough structure to obtain results.

The notion was translated to the context of topological equivalence relations by T. Giordano, I.F. Putnam and C.F. Skau in [GPS2004]: Affable equivalence relations are inductive limits of compact étale equivalence relations. In particular, those compact relations are finite. As T. Giordano, I.F. Putnam and C.F. Skau show, minimal actions on the Cantor set are affable, and in fact the converse is also true for minimal equivalence relations. Recently have been showed that minimal $\mathbf{Z}^2$-actions are also affable [GMPS2008, AGL2008].

On the other hand, affability can be thought as a topological version hyperfiniteness. This point of view allows us to use the ideas of [Series] and show that all transversally Cantor laminations of polynomial growth are affable, and hence, the transverse equivalence relation is orbit equivalent to an action of $\mathbf{Z}$.

[AGL2008]. Alcalde Cuesta, P. González Sequeiros and A. Lozano Rojo, Affability of euclidean tilings, Comptes Rendus Mathematique, 347 (2009), 947-952.

[GMPS2008] T. Giordano, H. Matui, I. F. Putnam and C.F. Skau, Orbit equivalence for Cantor minimal $\mathbf{Z}^2$-systems, J. Amer. Math. Soc., (2008).
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[GPS2004] T. Giordano, I.F. Putnam and C.F. Skau, Affable equivalence relations and orbit structure of Cantor dynamical systems, Ergodic Theory Dynam. Systems, 24 (2004), 441-476.

[Series1979] C.E. Series, Foliations of polynomial growth are hyperfinite, Israel J. Math., 34 (1979), 245-258.

Gael Megniez (Université de Bretagne Sud)

Title: Haefliger structures of codimension 1 made regular and minimal.

Abstract: A Haefliger structure is a singular foliation. W. Thurston's celebrated 1976 theorem regularized Haefliger structures of codimension 1 on compact manifolds through homotopy, under the obvious embedding condition. We give a new proof, which uses only elementary means, and not Mather's homology equivalence. Thurston's hole-filling methods remain the heart of the proof. Moreover, in dimensions 4 and more, the resulting foliation can be made minimal: all leaves are dense. e.g. the 5-sphere carries a minimal smooth foliation.

Yoshi Mitsumatsu (Chuo University)

Title: Turbulization of 2-dimensional foliations on 4-manifolds and tautness

Abstract:
A process of turbulizing a 2-dimensional foliation on 4-manifolds along a closed transversal is reivewed. One of particular features is that the process is deeply related to 3-dimensional algebraic Anosov flows. The tautness, especially the geometric tautness in the sense of Sullivan of the resultant foliations is discussed.

Shigeaki Miyoshi (Chuo University, Tokyo, Japan)

Title: A construction of a typical foliation on a 3-manifold

Abstract:
We show that every closed orientable 3-manifold has a smooth (C^\infty) codimension-one foliation. A typical foliation is the one whose every leaf has the contractible holonomy covering. A foliation is typical if and only if the classifying space of its holonomy groupoid is homotopy equivalent to the underlying manifold. D. Calegari constructed a typical foliation of class at most C^1 on every closed orientable 3-manifold. Our construction is based on the fact that the figure eight knot is universal, i.e., every closed orientable 3-manifold is a branched covering of the three sphere branched along the figure eight knot.

Hiraku Nozawa (École Normale Supérieure de Lyon)

Title: 5-dimensional K-contact geometry via the Morse theory on moment maps

Abstract:
A K-contact manifold is a contact manifold whose Reeb flow is a Riemannian foliation. Main examples are Sasakian manifolds, which include links of isolated singularities and toric contact manifolds. The dimension of closures of generic leaves of the Reeb flow of a K-contact manifold (M,α) is called the rank of (M, α). If dim M = 5, then the rank of (M, α) is equal to either of 1, 2 or 3. The geometry of K-contact 5-manifolds is well understood by toric geometry except the case of rank 2. On the other hand, a K-contact 5-manifold (M, α) of rank 2 has a nontrivial moment map which has rich information on the global structure of (M, α). In this talk, we discuss the following results i), ii), iii) and iv) obtained by the application of the Morse theory to the moment maps of closed K-contact 5-manifolds (M, α) of rank 2:

i) The isomorphism class of (M, α) is determined by its graph of isotropy data, which represent Morse theoretic data of the moment map,

ii) (M, α) is obtained from a lens space bundle over a closed surface by contact blowing up and down,

iii) (M, α) has a Sasakian metric,

iv) A combinatorial condition on the Reeb flow implies that (M, α) is toric.

Our main result is ii), which describes large difference from the geometry of symplectic 4-manifolds with hamiltonian S^S-actions. We also state some conjectures on Sasakian Einstein 5-manifold posed by physicists. The reference is arXiv:0907.0208.

Jorge Vitório Pereira (IMPA)

Title: The characteristic variety of a generic foliation

Abstract: The characteristic variety of holomorphic foliation by curves F is nothing more than the total space of its conormal bundle. The natural symplectic structure on the cotangent bundle of the ambient space induces a foliation by curves on the characteristic variety of F. This foliation is called the the first prolongation of F.

I will discuss a conjecture of Bernstein and Lunts which predicts that the first prolongation of an algebraic foliation on $\mathbf{C}^n$ leaves no algebraic subvarities invariant besides the zero section and subvarities of fibers over singular points.

Luis Pozo (Universidad Complutense de Madrid)

Title: Cohomology of horizontal forms

Abstract: We propose a new approach to the study of the cohomology of the complex of horizontal forms of a C^\infty foliation introduced by Vaisman. We give an explicit formula for an s-horizontal primitive of an s-horizontal closed form and we analyse the problem of representing a de Rham cohomology class by means of a horizontal closed form.

José Ignacio Royo Prieto (Universidad del País Vasco)

Title: Cohomology of S^3-actions and duality.
(Joint work with M. Saralegi Aranguren)

Abstract: We construct a Gysin sequence for any smooth S^3-action on a closed manifold M. An exotic term appearing in the Gysin sequence is an obstruction of the duality of the spectral sequence associated to the underlying Singular Riemannian Foliation.

Paul Schweitzer S.J. (PUC RIO)

Title: On the structure of homological Reeb components in codimension one foliations
(joint work with Fernando Alcalde and Gilbert Hector)

Abstract: We define a (homological) Reeb component to be a codimension one foliation on a compact manifold with boundary, such that the interior fibers over the circle with the interior leaves as fibers, the boundary components are also leaves, and there is a transverse orientation pointing inwards along the boundary. (This last condition is automatic if the boundary is connected.)

The classical Reeb component on the solid torus possesses a homotopical vanishing cycle (a non-contractible loop on a given leaf, which becomes contractible on nearby leaves), namely the meridian on the boundary torus. Similarly, every homological Reeb component has an analogous homological vanishing cycle, defined as follows. If the leaf dimension is p, then a homological vanishing cycle is a (p-1)-cycle on a leaf (or on a finite union of leaves) that is not homotopic to zero on any of these leaves, but can be isotoped onto every nearby leaf on one side, where it lies on a single leaf and becomes null-homotopic on the leaf. Every homological Reeb component has a canonical homological vanishing cycle that is unique up to sign, and every other vanishing cycle is an integrable multiple of the canonical one.

We give a general structure theorem for homological Reeb components and provide a variety of examples. For example, Whitehead's contractible open 3-manifold that is not homeomorphic to $\mathbf{R}^3$ can be a leaf in a 4-dimensional Reeb component. We also give partialconditions under which a Reeb component is fibered, i.e., obtained from a fibration over the circle with fiber a connected manifold C with non-empty boundary; the fibers are spun in the circle direction as they approach the boundary, so that the boundary components become leaves and the interior leaves are homeomorphic to the interior of C.

Vladimir Slesar (University of Craiova, Romania)

Title: A vanishing result for the $Spin^c$ Dirac operator defined along the leaves of a foliation

Abstract: We prove a Lichnerowicz type formula for this Dirac operator associated to an even dimensional distribution.

Dirk Toeben (Universität Köln, Germany)

Title: Equivariant Basic Cohomology of Riemannian Foliations

Abstract: The basic cohomology of a Riemannian foliation on a complete manifold with all leaves closed is the cohomology of the leaf space. In this paper we introduce various methods to compute the basic cohomology in the presence of both closed and non-closed leaves in the simply-connected case (or more generally for Killing foliations): We show that the total basic Betti number of the union C of the closed leaves is smaller than or equal to the total basic Betti number of the foliated manifold, and we give sufficient conditions for equality. If there is a basic Morse-Bott function with critical set equal to C we can compute the basic cohomology explicitly. Another case in which the basic cohomology can be determined is if the space of leaf closures is a simple, convex polytope.

Our results are based on Molino's observation that the existence of non-closed leaves yields a distinguished transverse action on the foliated manifold with fixed point set C. We introduce equivariant basic cohomology of transverse actions in analogy to equivariant cohomology of Lie group actions enabling us to transfer many results from the theory of Lie group actions to Riemannian foliations. The prominent role of the fixed point set in the theory of torus actions explains the relevance of the set C in the basic setting. This is a joint work with Oliver Goertsches.

Takashi Tsuboi (University of Tokyo)

Title: On the uniform perfectness of the group of diffeomorphisms

Abstract:
We show that the identity component $\mathrm{Diff}^r(M^m)_0$ of the group of C^r diffeomorphisms of a compact m-dimensional manifold M^m ($1\leq r\leq \infty$, $r\neq m+1$) is uniformly perfect for $m\neq 2, 4$, i.e., any element of $\mathrm{Diff}^r(M^m)_0$ can be written as a product of a bounded number of commutators. It is also shown that for a compact connected manifold M^m ($m\neq 2, 4$), the identity component $\mathrm{Diff}^r(M^m)_0$ of the group of C^r diffeomorphisms of M^m ($1\leq r\leq \infty$, $r\neq m+1$) is uniformly simple, i.e., for elements f and g of $\mathrm{Diff}^r(M^m)_0 \setminus \{id\}$, f can be written as a product of a bounded number of conjugates of g or g^{-1}.

Denis Volk

Title: Skew products with interval fiber

Abstract: Skew products with a topological Markov chain in base naturally appear when one attempts to apply the methods of classical dynamical systems to random dynamical systems. There is also a close connection between the skew products and partially hyperbolic dynamical systems on smooth manifolds.

Even for the fiber dimension equal to one, we are far from understanding what "typical" skew products look like. During the last 30 years there appeared several papers studying the skew products with a circle fiber. I will talk about the case when the fiber is an interval, and fiber maps are orientation-preserving diffeomorphisms onto its image.

In the work joint with V.Kleptsyn, we developed a theorem which gives us a complete* description of the dynamics of typical step skew products (fiber map depends only on a single symbol in the base sequence). For generic skew products, we obtained a similar result using an additional assumption of partial-hyperbolic nature.

*except some subset which projects onto zero measure set in the base

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Pawel Walczak (Uniwersytet Lodzki)

Title: Extrinsic geometric flows
(joint work with Vladimir Rovenski)

Abstract: We consider a foliated manifold (M, F) equipped with a fixed normal field N. Given a transformation F acting on symmetric (0,2)-tensors, we consider 1-parameter families (g_t) of Riemannian structures on M making N unit and orthogonal to F and satisfying along F the equation $dg_t/dt= F(b_t)$, b_t being the second fundamental form of the leaves of F with respect to g_t. Such a family (g_t) is called *extrinsic geometric flow* on (M,F). During this lecture we shall:
(1) discuss motivation for studying such flows,
(2) show local existence and uniqueness results,
(3) discuss some examples and
(4) list some problems concerning such flows.

References

[RW] V. Rovenski, P. Walczak, Extrinsic geometric flows on foliated manifolds, preprint, arXiv:1003.1607

Szymon. M. Walczak (Uniwersytet Lodzki)

Title: Hausdorff leaf spaces for foliations of codimension one

Abstract: The talk will present the results published in [W]. The topology of Hausdorff leaf spaces (briefly the HLS) for foliation of codimension one will be discussed. First, we recall the notion of the Hausdorff Leaf Spaces: Let (M,F,g) be a compact foliated Riemannian manifold. Let us set $\rho(L,L')$ equal to the infimum of the sums of the distances $dist(L_i,L_{i+1})$ over all finite sequences of leaves, $L_1 = L, L_2, \ldots, L_n = L'$. Let ~ be an equivalence relation in the space of leaves defined by $L \sim L'$ iff $\rho(L,L') = 0$. The corresponding quotient space has a metric induced by ρ. We call it the Hausdorff leaf space for the foliation F (briey the HLS), and we denote it by HLS(F). Let the codimension of F be equal to one. The HLSs associated with foliations obtained by basic constructions such as transversal and tangential gluing, spinning, turbulization and suspension will be shortly presented. After that the main results of [W] will be announced:

Theorem 1. Let (M,F, pr) be a foliated I-bundle. HLS(F) is isometric to a metric segment or a singleton.

Recall now that the metric graph G is the result of gluing of a set of a disjoint metric segments $E = \{E_i\}$ and points $V = \{v_i\}$ along an equivalence relation defined in the union of V and the set of the endpoints of the segments equipped with the length metric. A graph G is called finite if V and E are finite.

Theorem 2. HLS(F) of any codimension one foliation on a compact Riemannian manifold is isometric to a finite connected metric graph.

Theorem 3. For every finite connected metric graph G there exists a compact foliated Riemannian manifold (M,F,g) such that HLS(F) is isometric to G. Moreover, every leaf of F is proper.

References

[W] Sz. M. Walczak, Hausdorff Leaf Spaces for foliations of codimension one, to appear in the Journal of the Mathematical Society of Japan.
