Extended Modules

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(Joint work with Wolfgang Hassler)

Suppose R and S are local rings and $(R,\mathbf{m}) \to (S,\mathbf{n})$ is a flat local homomorphism of commutative local rings. Given a finitely generated S-module N, we say N is *extended* (from R) provided there is an R-module M such that $S \otimes_R M$ is isomorphic to N as an R-module. If such a module M exists, it is unique up to isomorphism, and it is necessarily finitely generated.

The $\mathbf{m}$-adic completion $R \to \widehat{R}$ and the Henselization $R \to R^{\mathrm{h}}$ are particularly important examples. One reason is that the Krull-Remak-Schmidt uniqueness theorem holds for direct-sum decompositions of finitely generated modules over a Henselian local ring. Indeed, failure of uniqueness for general local rings stems directly from the fact that some modules over the Henselization (or completion) are not extended. Understanding which R^{h}-modules are extended is the key to unraveling the direct-sum behavior of R-modules.

In this talk, I will survey criteria for an S-module to be extended, as well as criteria for a given module to be a direct summand of an extended module. This latter property is useful in questions concerning finite representation type.